Lý thuyết đồng điều và đối đồng điều checked 27/2

of (0, isT)-modules, where A->E is an inclusion map. If now the sequence

describes a second extension, then the two are said to be equivalent if there exists a (0, if)-isomorphism E « E* such that the diagram

……………………………..

is commutative.

của(G,K)mô-đun, trong đó $A \rightarrow E$là một ánh xạ co. Nếu bây giờ dãy

$$0 \to A \to E^* \to B \to 0$$

mô tả một mở rộng thứ hai, thì 2 mở rộng được gọi là tương đương nếu tồn tại một(G,K)-đẳng cấu $E \approx E^*$sao cho sơ đồ

Inclusion map: một số tài liệu dịch là "ánh xạ nhúng chìm"

$$\begin{tikzcd} & E \arrow[dr] \arrow[dd] & \\ A \arrow[ur] \arrow[dr] & & B \\ & E^* \arrow[ur] & \end{tikzcd}$$

giao hoán.

One particular extension of A by B can be obtained by taking for E the direct sum of A and B ; and this then determines a special class of equivalent extensions all of whose members will be said to split. Clearly the extension (10.9.7) splits if and only if A is a direct summand of E considered as (G, if)-module.

Một mở rộng đặc biệt của A bởi B có thể thu được bằng cách cho E là tổng trực tiếp của A và B, và như vậy điều này xác định một lớp đặc biệt củacác mở rộng tương đương mà tất cả các thành phần của nó sẽ chia tách .Rõ ràng mở rộng(10.9.7) chia tách khi và chỉ khi A là một hạng tử trực tiếp của Eđược xem như là (G,K)-mô-đun.

We shall now consider the problem of finding a convenient model for the classes of equivalent extensions of A by B. As a first step towards bringing in the cohomology theory of G, observe that if C and D are representation modules, then HornK(C,D) becomes a (0, iQ-module if we define cru by

(cru) c = <x{u((T~1c)} (ceC).   (10.9.8)

Bây giờ chúng ta sẽ xét bài toán tìm kiếm một mô hình thuận tiện cho các lớp của các mở rộng tương đương của A qua B. Với tư cách là một bước đầu tiên hướng tới việc đưa ra lý thuyết đối đồng điều của G, thấy rằng nếu C và D là các mô-đun đại diện, thì $Hom_K(C,D)$ trở thành một(G,K)-mô-đun nếu chúng ta định nghĩa σu bởi

$$(\sigma u)c = \sigma\{u(\sigma^{-1}c)\} \quad (c \in C). \qquad (10.9.8)$$

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[redacted]

[redacted]

[redacted]

$$0 \rightarrow \mathrm{Hom}_K(B,A) \rightarrow \mathrm{Hom}_K(B,E) \rightarrow \mathrm{Hom}_K(B,B) \rightarrow 0 \qquad (10.9.9)$$

[redacted]

[redacted]

$$\Delta : [\mathrm{Hom}_K(B, B)]^G \to H^1\{G, \mathrm{Hom}_K(B, A)\}.$$

[redacted]

$$\Delta(i_B) \in H^1\{G, \mathrm{Hom}_K(B, A)\}.$$

[redacted]

[redacted]

$$0\to \mathrm{Hom}_K(B,A)\to \mathrm{Hom}_K(B,E)\to \mathrm{Hom}_K(B,B)\to 0$$

[redacted]

$$\mathrm{Im}\,(\sigma\phi-\phi)\subseteq A$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$0 \to A \to E \to B \to 0$ [redacted] $0 \to A \to E' \to B \to 0$

[redacted]

$$H^1\{G, \mathrm{Hom}_K(B, A)\}.$$

[redacted]

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[redacted]

[redacted]

[redacted]

$$\sigma(a+\phi(b)) = \sigma a + [\sigma\phi](\sigma b) = \sigma a + [\sigma\phi - \phi](\sigma b) + \phi(\sigma b).$$

[redacted]

$$\begin{aligned}\sigma a + [\sigma\phi - \phi](\sigma b) + \phi'(\sigma b)& \\ &= \sigma a + [\sigma\phi' - \phi'](\sigma b) + \phi'(\sigma b) = \sigma(a+\phi'(b)).\end{aligned}$$

[redacted]

[redacted]

[redacted]

[redacted]

$$\sigma h_\tau - h_{\sigma\tau} + h_\sigma = 0 \quad (\sigma, \tau \in G), \tag{10.9.10}$$

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[redacted]

[redacted]

[redacted]

$$\sigma(a, b) = (\sigma a + h_\sigma(\sigma b), \sigma b).$$

[redacted]

$$\begin{aligned}\sigma\{\tau(a, b)\} = \sigma\{\tau a + h_\tau(\tau b), \tau b\} &= \{\sigma\tau a + [\sigma h_\tau]\,(\sigma\tau b) + h_\sigma(\sigma\tau b), \sigma\tau b\} \\ &= \{\sigma\tau a + h_{\sigma\tau}(\sigma\tau b), \sigma\tau b\} = (\sigma\tau)\,(a, b).\end{aligned}$$

[redacted]

[redacted]

$$(\sigma\phi - \phi)\,b = \sigma(\phi(\sigma^{-1}b)) - \phi(b) = \sigma(0, \sigma^{-1}b) - \phi(b)$$
$$= (h_\sigma(b), b) - (0, b) = (h_\sigma(b), 0).$$

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[redacted]

$$H^1\{G, \mathrm{Hom}_K(B, A)\} = X\,(\text{say}) = 0,$$

[redacted]

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[redacted]

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[redacted]

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[redacted]

[redacted]

$$\sigma a - a = -qf(\sigma)$$

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[redacted]

[redacted]

[redacted]

[redacted]

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[redacted]

[redacted]

[redacted]

[redacted]

$$\sigma f(\tau,\rho) - f(\sigma\tau,\rho) + f(\sigma,\tau\rho) - f(\sigma,\tau) = 0. \qquad (10.10.1)$$

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[redacted]

[redacted]

$$f(\sigma,\tau) = \sigma\phi(\tau) - \phi(\sigma\tau) + \phi(\sigma) \qquad (10.10.2)$$

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[redacted]

[redacted]

$$(a_1 a_2)^\sigma = a_1^\sigma a_2^\sigma, \quad (a^\sigma)^\tau = a^{\tau\sigma}, \quad a^1 = a, \qquad (10.10.3)$$

[redacted]

[redacted]

[redacted]

$$a^*_{\sigma,\tau} = a'_{\sigma,\tau} a''_{\sigma,\tau}.$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$a^\sigma_{\tau,\rho} = \frac{a_{\sigma\tau,\rho} a_{\sigma,\tau}}{a_{\sigma,\tau\rho}} \quad (\sigma, \tau, \rho \in G), \qquad (10.10.4)$$

[redacted]

$$a_{\sigma,\tau} = \frac{\alpha_\tau^\sigma \alpha_\sigma}{\alpha_{\sigma\tau}} \quad (\sigma, \tau \in G). \qquad (10.10.5)$$

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[redacted]

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[redacted]

[redacted]

$$\begin{tikzcd}
 & \Pi \arrow[dr, "\chi"] \arrow[dd] & \\
A \arrow[ur] \arrow[dr] & & G \\
 & \Pi^* \arrow[ur, "\chi^*"'] &
\end{tikzcd}$$

[redacted]

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[redacted]

[redacted]

$$\pi'_\sigma a \pi'^{-1}_\sigma = \alpha \pi_\sigma a \pi_\sigma^{-1} \alpha^{-1} = \pi_\sigma a \pi_\sigma^{-1},$$

[redacted]

$$a^\sigma = \pi_\sigma a \pi_\sigma^{-1} \quad (a \in A,\ \sigma \in G), \tag{10.10.6}$$

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[redacted]

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[redacted]

[redacted]

[redacted]

$\chi(\pi_\sigma \pi_\tau) = \sigma\tau = \chi(\pi_{\sigma\tau})$, [redacted]

$$\pi_\sigma \pi_\tau = a_{\sigma,\tau} \pi_{\sigma\tau},$$

[redacted]

$$(\pi_\sigma \pi_\tau) \pi_\rho = a_{\sigma,\tau} \pi_{\sigma\tau} \pi_\rho = a_{\sigma,\tau} a_{\sigma\tau,\rho} \pi_{\sigma\tau\rho},$$

[redacted]

$$\pi_\sigma(\pi_\tau \pi_\rho) = \pi_\sigma a_{\tau,\rho} \pi_{\tau\rho} = \pi_\sigma a_{\tau,\rho} \pi_\sigma^{-1} \pi_\sigma \pi_{\tau\rho}$$

$$= a^\sigma_{\tau,\rho} a_{\sigma,\tau\rho} \pi_{\sigma\tau\rho};$$

[redacted] $a_{\sigma,\tau} a_{\sigma\tau,\rho} = a^\sigma_{\tau,\rho} a_{\sigma,\tau\rho}$ [redacted]

[redacted]

[redacted]

$a_{\sigma,\tau} = a^*_{\sigma,\tau}$ [redacted] $a_{\sigma,\tau} = a^*_{\sigma,\tau}$ [redacted] $\alpha\pi_\sigma \to \alpha\pi^*_\sigma$, [redacted]

[redacted]

$\alpha = \alpha a_{1,1}^{-1}\pi_1 = \alpha a_{1,1}^{-1}\pi_1^*$ [redacted]

[redacted]

[redacted]

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[redacted]

[redacted] $a_{\sigma,\tau} = a^*_{\sigma,\tau}$ [redacted]

[redacted] $\overline{\pi}_\sigma = \alpha_\sigma \pi_\sigma$, [redacted]

$$\overline{\pi}_\sigma \overline{\pi}_\tau = \alpha_\sigma \pi_\sigma \alpha_\tau \pi_\tau = \alpha_\sigma \pi_\sigma \alpha_\tau \pi_\sigma^{-1} \pi_\sigma \pi_\tau = \alpha_\sigma \alpha_\tau^\sigma a_{\sigma,\tau} \pi_{\sigma\tau},$$

[redacted] $\overline{\pi}_\sigma \overline{\pi}_\tau = \overline{a}_{\sigma,\tau} \overline{\pi}_{\sigma\tau} = \overline{a}_{\sigma,\tau} \alpha_{\sigma\tau} \pi_{\sigma\tau}$,

[redacted]

$$\overline{a}_{\sigma,\tau} = a_{\sigma,\tau} \frac{\alpha_\tau^\sigma \alpha_\sigma}{\alpha_{\sigma\tau}}.$$

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$$a_{\sigma,1} = 1_A = a_{1,\sigma} \quad (\sigma \in G). \qquad (10.10.7)$$

[redacted]

$$(\alpha, \sigma)(\beta, \tau) = (\alpha\beta^{\sigma}a_{\sigma,\tau}, \sigma\tau),$$

[redacted]

[redacted]
$$[(\alpha, \sigma)(\beta, \tau)](\gamma, \rho) = (\alpha\beta^{\sigma}\gamma^{\sigma\tau}a_{\sigma,\tau}a_{\sigma\tau,\rho}, \sigma\tau\rho),$$
$$(\alpha, \sigma)[(\beta, \tau)(\gamma, \rho)] = (\alpha\beta^{\sigma}\gamma^{\sigma\tau}a^{\sigma}_{\tau,\rho}a_{\sigma,\tau\rho}, \sigma\tau\rho),$$

[redacted] $a_{\sigma,\tau}a_{\sigma\tau,\rho} = a^{\sigma}_{\tau,\rho}a_{\sigma,\tau\rho}.$ [redacted]

[redacted]

[redacted] (10.10.7), $(\alpha,\sigma)e = (\alpha,\sigma) = e(\alpha,\sigma),$ [redacted]

$$(\alpha,1)(\alpha^{-1},1) = e,$$

[redacted] $(\alpha,\sigma)(1_A,\sigma^{-1}) = (\beta,1)$ [redacted] $(1_A,\sigma^{-1})(\alpha,\sigma) = (\gamma,1)$ [redacted]

$$(\alpha,\sigma)(1_A,\sigma^{-1})(\beta^{-1},1) = e = (\gamma^{-1},1)(1_A,\sigma^{-1})(\alpha,\sigma).$$

[redacted]

$$(1_A,\sigma)(\alpha,1)(1_A,\sigma)^{-1} = (\alpha^{\sigma},\sigma)(1_A,\sigma)^{-1} = (\alpha^{\sigma},1)(1_A,\sigma)(1_A,\sigma)^{-1}$$
$$= (\alpha^{\sigma},1),$$

[redacted]

$$(1_A, \sigma)\,(1_A, \tau) = (a_{\sigma,\tau}, \sigma\tau) = (a_{\sigma\tau}, 1)\,(1_A, \sigma\tau),$$

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[redacted]

$$\phi(a) = a \quad (\text{all}\, a \in A) \quad \text{and} \quad \chi\phi = \chi. \qquad (10.10.8)$$

[redacted]

$$\phi_\alpha(\pi) = \alpha\pi\alpha^{-1} \quad (\pi \in \Pi);$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\phi(\pi'_\sigma) = \alpha\phi(\pi_\sigma) = a\alpha\pi_\sigma = a\pi'_\sigma;$$

[redacted]

$$\phi(\pi_\sigma) = a_\sigma\pi_\sigma \quad (\sigma \in G). \qquad (10.10.9)$$

[redacted]

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[redacted]

[redacted]

$$\phi(\pi_\sigma \pi_\tau) = \phi(\pi_\sigma)\,\phi(\pi_\tau) = a_\sigma \pi_\sigma a_\tau \pi_\tau = a_\sigma a_\tau^\sigma \pi_\sigma \pi_\tau,$$

[redacted]

[redacted]

$$\phi'(\alpha\pi_\sigma) = a'_\sigma \alpha\pi_\sigma \quad (\alpha \in A).$$