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## Chapter 3 Physics and Overview of Electromagnetic Scattering

J. F. Shaeffer **checked**

### 3.1 INTRODUCTION

The objective of this chapter is to introduce the concept of radar cross section and the fundamentals of electromagnetic scattering in an overview fashion so that the reader may then delve into the remainder of the book. The topics to be presented are

- Terms: The definition of radar cross section from IEEE, an intuitive derivation, the polarization scattering matrix for linear polarization and its conversion to circular polarization, and the definition of total cross section and extinction cross section and the forward scattering theorem;
- Fundamental physical processes of electromagnetic scattering: Electromagnetic wave fundamentals, induced charges and currents, field lines attached to charges, near, intermediate and far fields, solenoidal and conservative fields, and the concepts for scattered, incident, and total field;
- Scattering regimes: The low-frequency Rayleigh region with induced-dipolelike scattering, the resonant region with attached surface

## Chương 3 Đặc Tính Vật Lý và Tổng Quan Về Tán Xạ Điện Từ

JF Shaeffer

### 3.1 GIỚI THIỆU

Mục tiêu của chương này nhằm giới thiệu khái niệm về tiết diện radar và các nguyên tắc cơ bản của quá trình tán xạ điện từ ở dạng tổng quan để sau này người đọc có thể đi sâu vào phần còn lại của sách. Chúng tôi sẽ đề cập đến các chủ đề

- Các thuật ngữ: Định nghĩa tiết diện radar của IEEE, một cách hình dung trực quan, ma trận tán xạ phân cực của chế độ phân cực tuyến tính và chuyển đổi sang dạng phân cực tròn của nó, và định nghĩa tiết diện toàn phần và tiết diện suy hao và định lý tán xạ thuận (tán xạ về phía trước);
- Các quá trình vật lý cơ bản của hiện tượng tán xạ điện từ: nguyên tắc cơ bản về sóng điện từ, điện tích và dòng điện cảm ứng, các đường sức của điện tích, các trường gần, trung bình và xa, các trường Sôlênôit và trường bảo toàn, và các khái niệm về trường tán xạ, trường tới, và trường toàn phần;
- Các chế độ tán xạ: Vùng Rayleigh tần số thấp cùng với tán xạ kiểu lưỡng cực cảm ứng, vùng cộng hưởng kèm theo

wave scattering; and the high-frequency optics region with the concepts of individual scattering centers; opticslike specular, end-region, and diffraction scattering mechanisms; phasor addition as how various scattering mechanisms sum to form a total scattered field; and the concepts for coherent and incoherent sums of individual scattering centers;

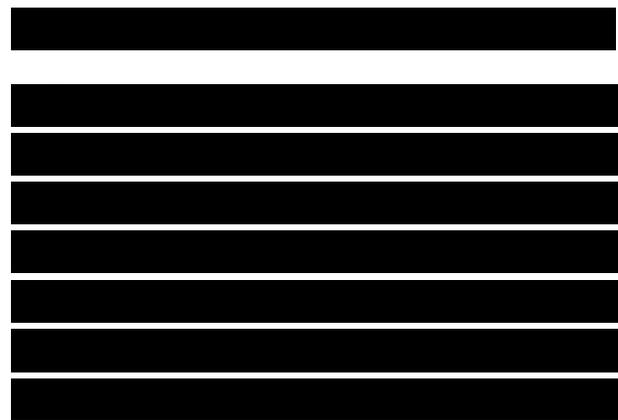
- Electromagnetic theory: Field quantities and their sources; Maxwell's equations in differential and integral form; vector and scalar potentials as sources for solenoidal and conservative field components; wave equation and characteristic solutions; waves at boundaries; reflection, transmission, and absorption coefficients; Fresnel reflection coefficients; EM wave formalism compared to transmission line theory; surface current point of view; and the Stratton-Chu integral equation formulation of Maxwell's equations with currents and charges as field sources.

### 3.2 RADAR CROSS SECTION DEFINITION

Radar cross section is a measure of power scattered in a given direction when a target is illuminated by an incident wave. RCS is normalized to the power density of the incident wave at the target so that it does not depend on the distance of the target from the illumination source. This removes the

tán xạ sóng bề mặt và vùng quang học tần số cao cùng với khái niệm về các tâm tán xạ riêng biệt; các cơ chế tán xạ gương kiểu quang học, end-region, và nhiễu xạ; cộng phasor nhằm mục đích tổng hợp các cơ chế tán xạ khác nhau và xây dựng trường tán xạ toàn phần, và các khái niệm tổng kết hợp và không kết hợp của các tâm tán xạ riêng biệt;

- Lý thuyết điện từ: Các đại lượng trường và nguồn gốc của chúng, các phương trình Maxwell ở dạng vi phân và tích phân; thể vector và vô hướng với tư cách với tư cách là các nguồn của thành phần trường Sôlênôit và trường bảo toàn; phương trình sóng và các nghiệm đặc trưng; sóng tại các biên, hệ số phản xạ, truyền qua và hấp thụ, hệ số phản xạ Fresnel; Hình thức luận sóng EM so với lý thuyết đường truyền, quan điểm dòng điện bề mặt, và dạng phương trình tích phân Stratton-Chu của các phương trình Maxwell với các dòng và các điện tích đóng vai trò là nguồn của trường.

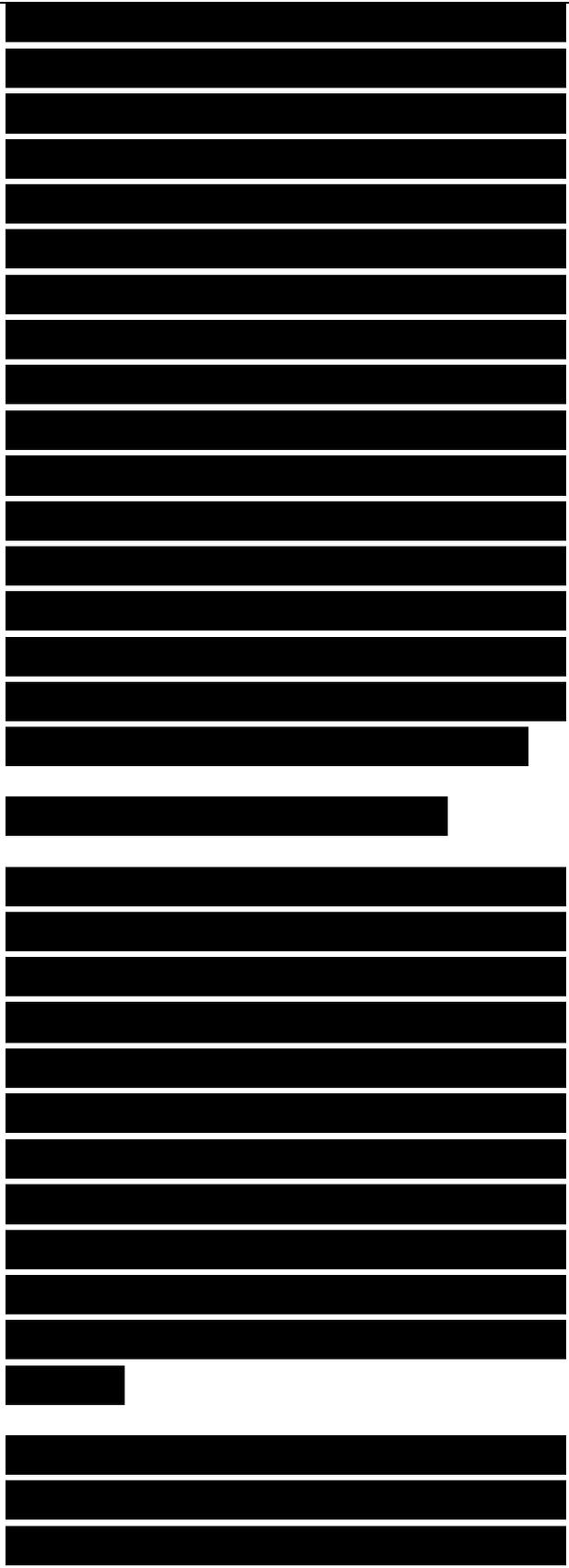


effects of the transmitter power level and distance to target when the illuminating wave decreases in intensity due to inverse square spherical spreading. RCS is also normalized so that inverse square fall-off of scattered intensity due to spherical spreading is not a factor so that we do not need to know the position of the receiver. RCS has been defined to characterize the target characteristics and not the effects of transmitter power, receiver sensitivity, and the position of the transmitter or receiver distance. Another term for RCS is echo area.

### 3.2.1 IEEE RCS Definition

The IEEE dictionary of electrical and electronics terms [1] defines RCS as a measure of reflective strength of a target defined as  $4\pi$  times the ratio of the power per unit solid angle scattered in a specified direction to the power per unit area in a plane wave incident on the scatterer from a specified direction. More precisely, it is the limit of that ratio as the distance from the scatterer to the point where the scattered power is measured approaches infinity:

where  $E_{scat}$  is the scattered electric field and  $E_{inc}$  is the field incident at the target. Three cases are distinguished:



monostatic or backscatter, forward scattering, and bistatic scattering.

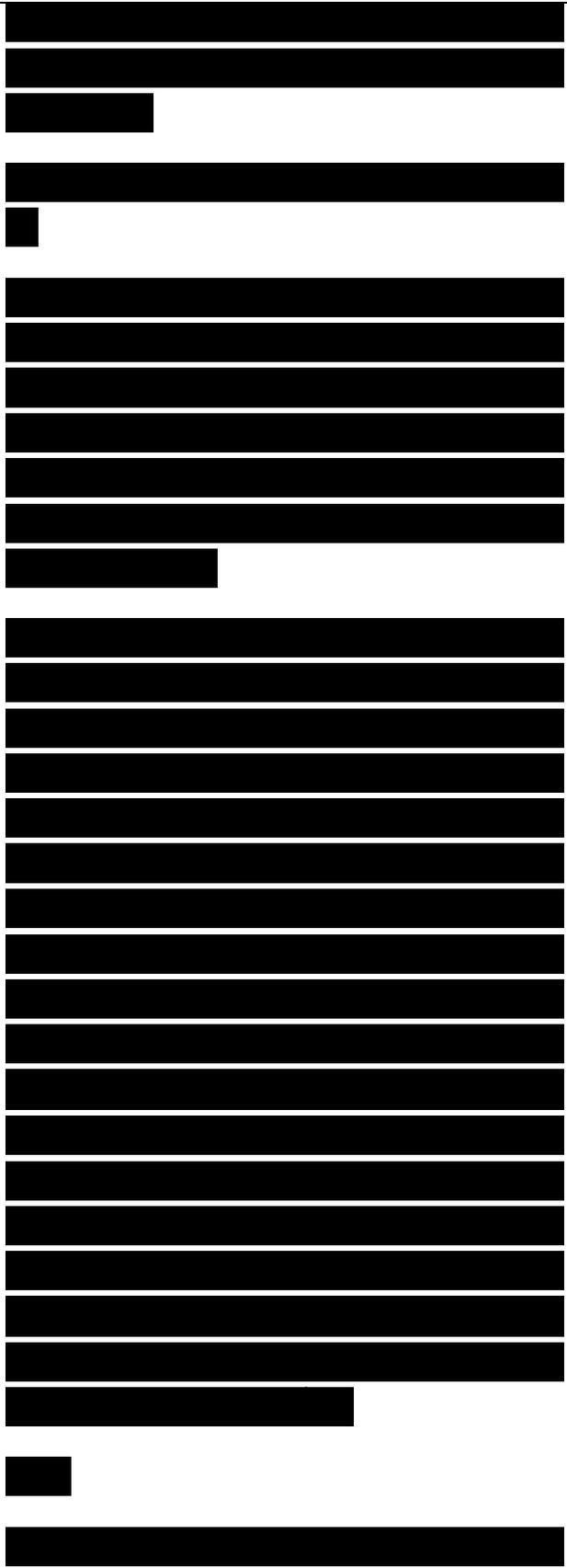
### 3.2.2 Intuitive Derivation for Scattering Cross Section

A formal cross section may be defined for the energy that is scattered, absorbed, removed from the incident wave, and the total cross section. The scattered energy is of greatest practical interest because it represents the energy available for detection.

The formal IEEE definition for RCS can be made more intuitive from the following derivation, Figure 3.1. Let the incident power density at scattering target from a distant radar be  $P$ ,  $W/m^2$  (which automatically removes from the definition transmitter power and inverse square intensity fall-off). The amount of power intercepted by the target is then related to its cross section  $\sigma$ , with units of area, so that the intercepted power is  $(\sigma P)$   $W$ . This intercepted power is then either reradiated as the scattered power or absorbed as heat. Assume for now that it is reradiated as scattered power uniformly in all  $4\pi$  sr of space so that the scattered power density, watts/meter<sup>2</sup>, is given by

(3.2)

We then solve (3.2) for  $\sigma$  and consider



that the distance  $R$  is far from the target to avoid nearfield effects:

RCS is therefore fundamentally a ratio of scattered power density to incident power density. The power or intensity of an EM wave is proportional to the square of the electric or magnetic field, so RCS can be expressed as

Figure 3.1. Intuitive definition for radar cross section.

(3.4)

because in the far field either  $E$  or  $H$  is sufficient to describe the EM wave.

The unit for cross section  $\sigma$  is area, usually in square meters, or may be nondimensional by dividing by wavelength squared,  $\lambda^2$ .

This definition is made more recognizable by examination of the basic radar range equation for power received by the radar,  $P_r$ , in terms of transmitted, scattered, and received power:

(3.5)

The first term in the numerator is the power density at the target from the transmitter. This term has units of watts per meter<sup>2</sup>. This incident power flux is multiplied by the cross section (area) and represents the power reflected back toward the receiver. When this is divided by the return path spherical spreading, we obtain the power density

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

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[REDACTED]

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at the receiver for capture by the receiving antenna effective area  $A_r$ .

Radar cross section is a function of

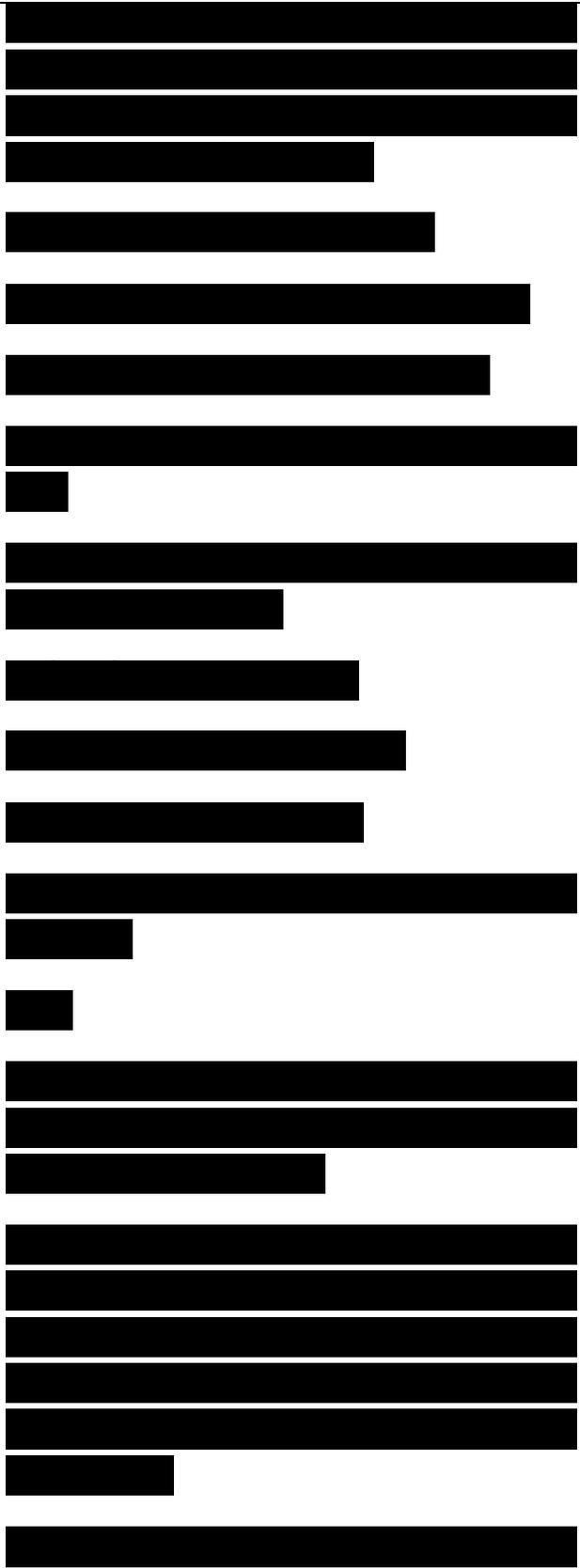
- Position of transmitter relative to target;
- Position of receiver relative to target;
- Target geometry and material composition;
- Angular orientation of target relative to transmitter and receiver;
- Frequency or wavelength;
- Transmitter polarization;
- Receiver polarization.

The general notation for indicating polarization and angle functionality is (3.6)

where  $t$  and  $r$  refer to transmitter and receiver polarization, typically horizontal or vertical, and angular coordinates.

Bistatic cross section is for the case when the transmitter and receiver are at different locations, Figure 3.2, so that (3.6) applies; that is, angular location of target relative to transmitter and receiver must be specified.

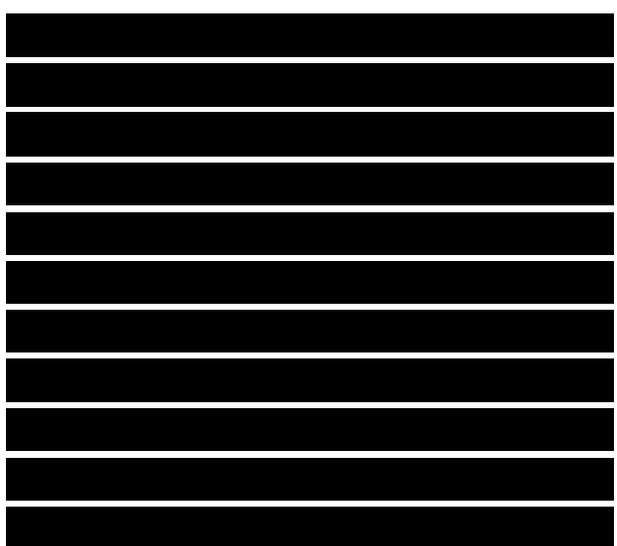
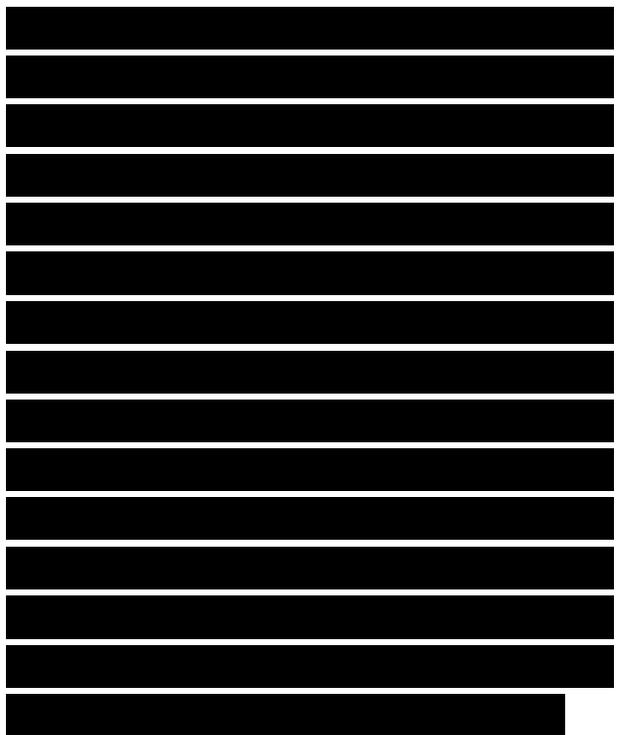
Forward cross section is the measure of



scattered power in the forward direction; that is, in the same direction as the incident field. This forward scattered power is usually  $180^\circ$  out of phase with the incident field so that when added to the incident field a shadow region is formed behind the scattering object.

Monostatic or backscatter cross section is the usual case of interest for most radar systems where the receiver and transmitter are collocated, oftentimes using the same antenna for transmitting and receiving, Figure 3.2. In this case only one set of angular coordinates is needed. Most experimental measurements are of backscatter cross section. Analytical RCS predictions, however, are much easier to do for bistatic cross section, with the illumination source fixed and the receiver position moved. One must be careful about analytical RCS predictions as to just which quantity is being presented.

Radar cross section of a target may also be a function of the pulse width  $T$  of the incident radiation. When  $T$  is large enough,  $T > 2L/c$ , where  $L$  is the body size and  $c$  the speed of light, the entire target is illuminated at once. This is the usual case for microsecond pulsewidths that have a spatial extent of 1000 ft or more. This is loosely equivalent to the target being illuminated by a continuous wave at a specific frequency, CW illumination. This is known as long-pulse illumination and is the usual

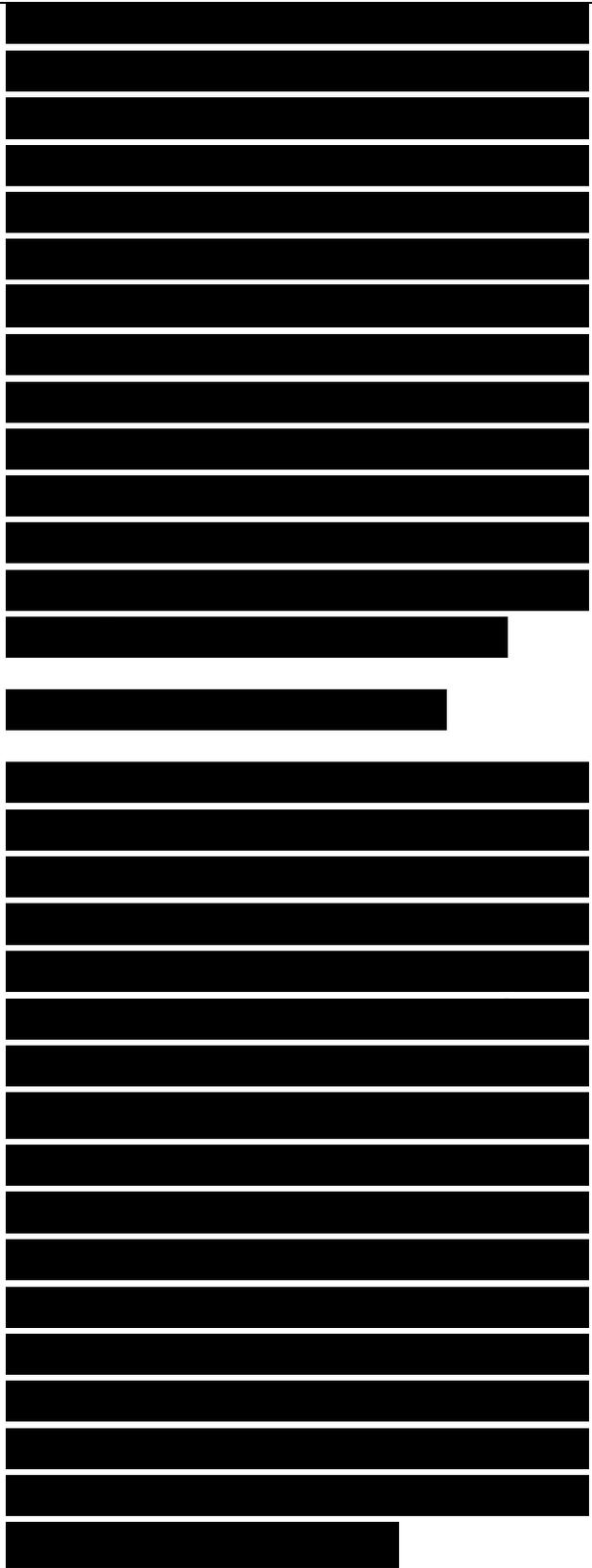


measurement case. When very short transmitter pulses are used, such as nanosecond pulses with a spatial extent of only several feet,  $T < 2L/c$ , then each scatterer on the target contributes independently to the return. In this case the RCS is a collection of individual scattering returns separated in time. Short-pulse radars (or their wide bandwidth equivalent) are often used to identify these scattering centers on complex targets.

#### RCS Customary Notation

The units for radar cross section are square meters. This does not necessarily relate to the physical size of a target. Although it is generally true that larger physical targets have larger cross sections (e.g., the optical front face reflection for a sphere is proportional to its projected area,  $\sigma_{sphere} = \pi r^2$ ), not all RCS scattering mechanisms are related to size as is shown in the hierarchy of scattering table. Typical values of RCS can span  $10^{-5}$  m<sup>2</sup> for insects to  $10^{+6}$  m<sup>2</sup> for large ships. Due to the large dynamic range of RCS, a logarithmic power scale is most often used with the reference value of  $\sigma_{ref} = 1$  m<sup>2</sup>:

(3.7)

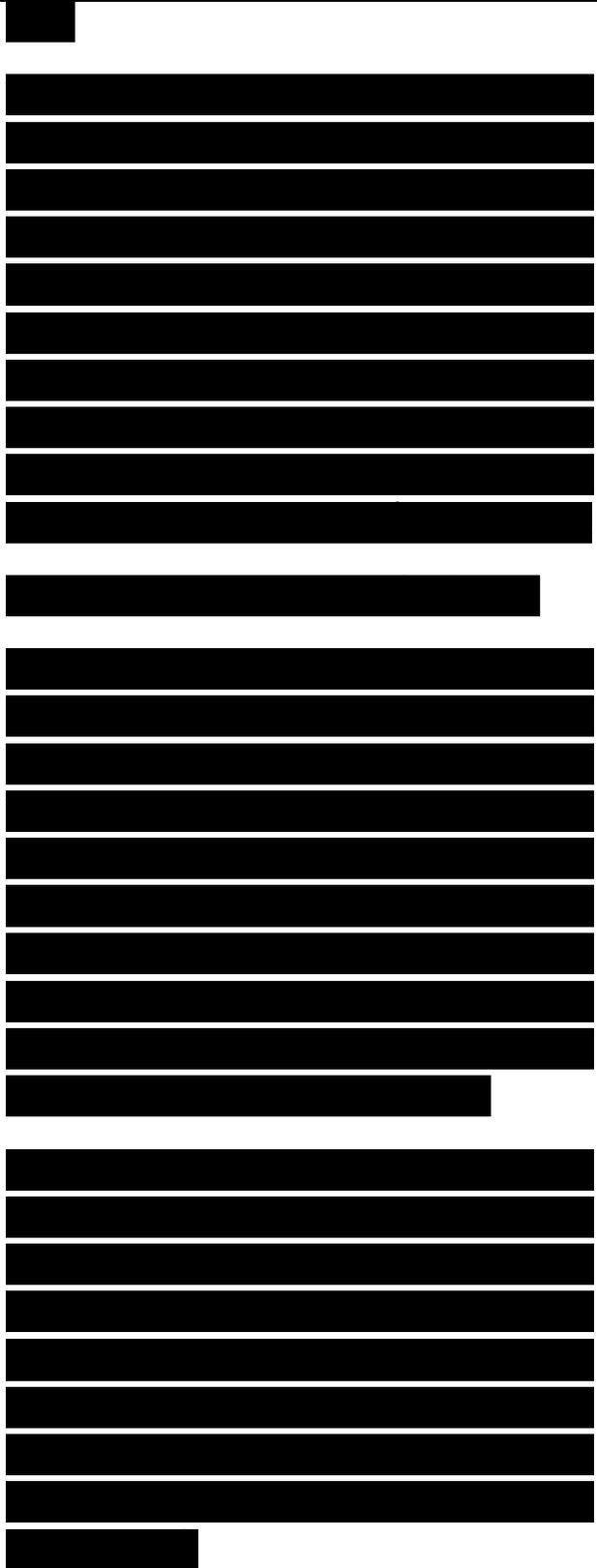


Two notations are used. The dBsm notation is customary within the academic, government, and industrial communities. The dBm<sup>2</sup> notation is less used, typically in radar system design literature. A comparison of the square meter and dBsm scales is shown in Figure 3.3. It is noted that 1 m<sup>2</sup> corresponds to 0 dBsm with fractional values having negative dBsm values; for example, 0.01 m<sup>2</sup> = — 20 dBsm.

### 3.2.3 Other Cross-Section Concepts

The cross-section concept defined above is for the power density scattered by a target in a given direction. As such it is our working definition because it represents or defines the power that may eventually be radiated back to a radar receiving antenna for possible detection. Often this cross section is referred to as the differential scattering cross section, as it gives the angular distribution of scattered power.

Several other scattering definitions may also be given. They are for power that is absorbed by a target, for the total power removed from the incident field, for the total power scattered by a target, and the forward scatter theorem. These additional concepts not often used in practice.



### Absorption Cross Section

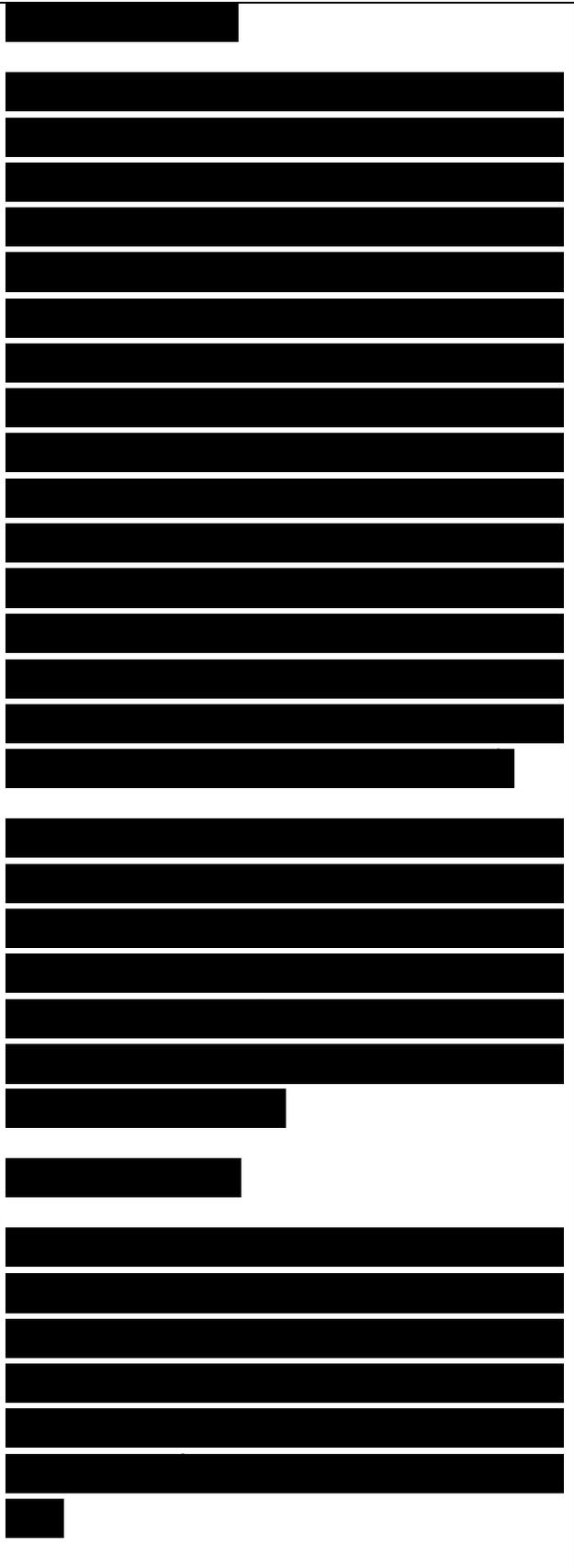
A scattering target may also absorb some of the incident EM wave power in addition to scattering. The absorption cross section is a measure of the absorbed incident power. Perfectly conducting targets do not absorb power as the resistivity is identically zero. They can only scatter. However, nonperfect conductor targets, such as those with absorbing materials, can turn some of the incident energy into heat. This energy of course is then not available for reradiation. The absorption cross section is defined as the amount of power absorbed by the target, in watts, normalized to the incident power density, in watts/meter<sup>2</sup>:

which depends on only transmitter location angular coordinates. The amount of power absorbed by a target may be specified in terms of currents and resistivities of the target and may be computed from analytical models, but otherwise it is difficult to determine.

### Extinction Cross Section

Power scattered and/or absorbed by a target is removed from the incident EM wave. Total power removed by virtue of scattering and absorption, in watts, normalized to the incident power density, in watts/meter<sup>2</sup>, is defined as the extinction cross section:

— power removed by scattering and absorption (W) e incident power



density (W/m<sup>2</sup>) (3.9)

= or + a-a

The extinction cross section is equal to the sum of the total scattering cross section, defined below, and the absorption cross section.

### Total Cross Section

The total scattering cross section or is a measure of the total power scattered by a target in all 4π sr spatial directions:

total scattered power (W),

$$P_{sc} = I_0 \sigma_{sc} \quad (3.10)$$

incident power density (W/m<sup>2</sup>)

It is formally defined by integrating the scattering (differential) cross section  $\frac{d\sigma}{d\Omega}$  over all spatial directions:

$$\sigma_{sc} = \int_{4\pi} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta \, d\phi \quad (3.11)$$

4π sr } 4π sr

This is also the 4-π steradian spatial average cross section. If  $\frac{d\sigma}{d\Omega}$  were constant over all spatial directions (a physical impossibility), then  $\sigma_{sc} = a$ . The total cross section has the physical interpretation of an area normal to the incident EM wave that intercepts an amount of incident power equal to the scattered power.

The usual scattering cross section (differential) then may also be defined in terms of the total cross section  $\sigma_T$ :

$$\sigma_r = \frac{4\pi}{k^2} \text{Im} \{f(0)\} \quad (3.12)$$

where we see why the term differential is applied; that is, it gives the amount of scattered power as a function of spatial coordinates.

### Forward-Scattering Theorem

The electric field scattered in the forward direction, when added to the incident field forms a shadow behind the target. (The forward-scattered field is  $180^\circ$  out of phase with the incident field, so addition actually means subtraction.) The darkness of this shadow is a measure of how much power was removed from the incident EM wave; that is, the greater the scattering the greater is the forward scatter and the darker is the shadow. The forward-scatter theorem relates the total cross section, which is the power removed from the incident wave by scattering, to the forward-scattered field. The explicit form is proportional to the imaginary part of the scattering amplitude  $F$  evaluated in the forward direction, written as [2,3]

$$(3.13)$$

where we have used the standard expression for the differential cross section defined in terms of the scattering amplitude function  $F(\theta, \phi)$ :

$$\sigma(\theta, \phi) = |f(\theta, \phi)|^2$$

Therefore the total power removed from the incident wave is related to the field scattered in the forward direction.

Each of these concepts for EM scattering has equivalent analogs for acoustic and particle physics scattering.

### 3.2.4 Polarization Scattering Matrix

Radar cross section, as a scalar number, is a function of the polarization of the incident and received wave. A more complete description of the interaction of the incident wave and the target is given by the polarization scattering matrix (PSM), which relates the scattered electric field vector  $E_s$  to the incident field vector  $E_i$ , component by component. In matrix notation, this is

As  $E$  can be decomposed into two independent directions or polarizations, because there is no component in the direction of propagation  $k$ , the polarization scattering matrix  $S$  is a  $2 \times 2$  complex matrix:

where  $E_s$  and  $E_i$  are the scattered and incident fields, each with independent vector components  $E_{s1}$  and  $E_{s2}$ . The components of  $S$  are related to the square root of cross section

(3.17)

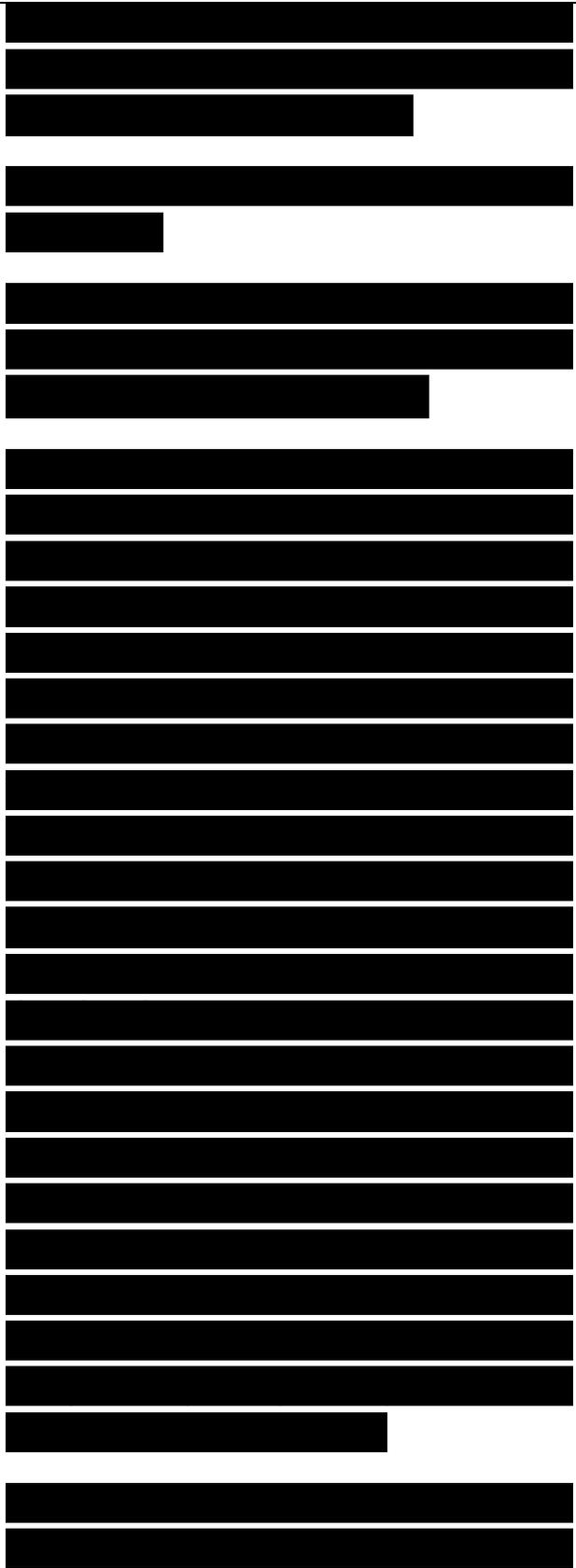
where we recognize  $V_{\theta}$  as a complex

number that has amplitude as well as phase. The radar received voltage,  $V_r$ , depends on the polarization of the receiver,  $n_r$ , by  
and on the polarization of the transmitted wave by

where  $a$  and  $f_3$  are the transmitted components of each polarization along the directions of  $E_1$  and  $E_2$ , respectively.

The scattering matrix is specified by eight scalar quantities, four amplitudes, and four phases. One phase angle is arbitrary and used as a reference for the other three. If the radar system is monostatic (backscatter), then  $S_{12} = S_{21}$  and  $S$  can then be specified by five quantities. If we had a coherent radar that transmitted and received two orthogonal polarizations, then the scattering matrix could be determined for a given aspect ( $\theta, \phi$ ) at frequency  $f$ . For a given target, aspect angle and frequency, we can extract no more signal information than that contained in the scattering matrix. The PSM approach to scattering is discussed by Huynen [4] who considers the eigenvalues and eigenvectors of the scattering matrix functions of target size, orientation, symmetry, double bounce polarization, and characteristic angle. Such information can be useful for target identification.

The PSM matrix can be defined for linear or circular polarization. Typical linear polarization directions are

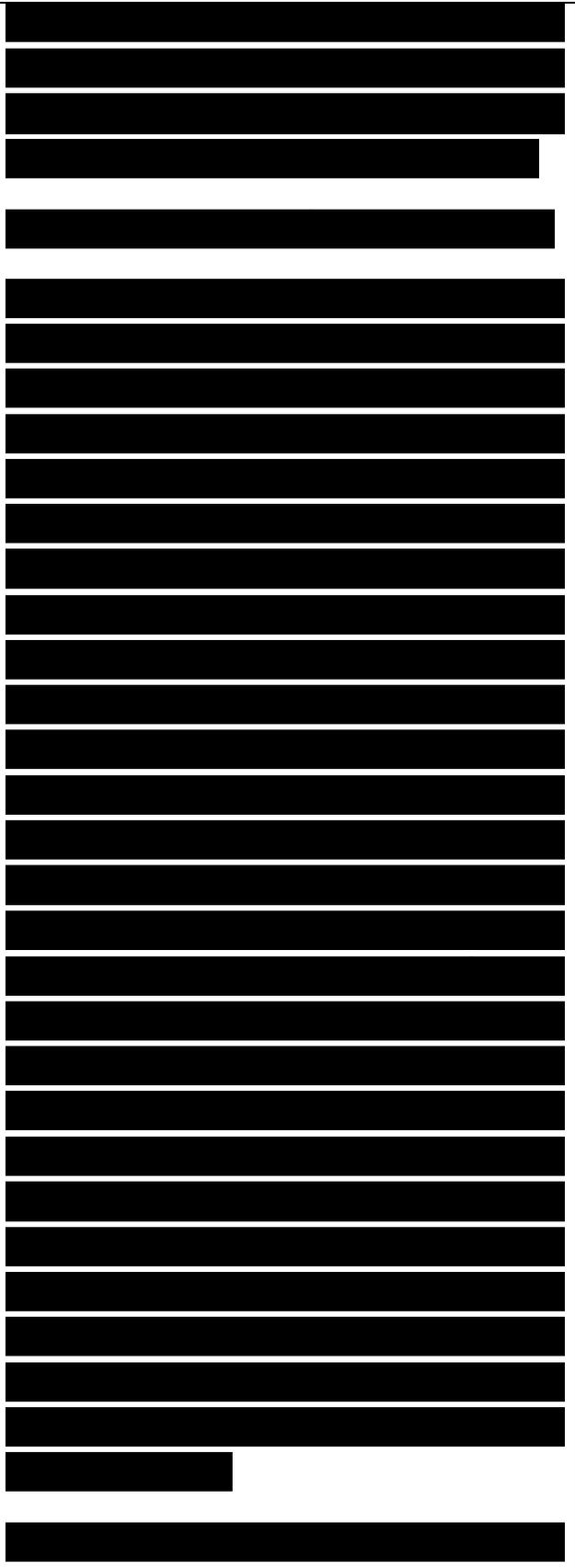


horizontal and vertical for experimental work and  $\theta$  and  $\phi$  spherical directions for analytical work.

### Scattering Matrix for Circular Polarization

In circular polarization, the electric field vector rotates in the plane perpendicular to propagation. The two independent directions then correspond to right- and lefthand rotation defined as clockwise or counterclockwise when the wave is viewed by a person looking at the wave going away from the observer, Figure 3.4. This is the IEEE definition for circular polarization (i.e., righthand polarization), the electric field vector rotates counterclockwise in time for an approaching wave and clockwise for a receding wave; for a lefthand polarization, the electric field vector rotates clockwise for an approaching wave and counterclockwise for a receding wave, [5]. Linear polarization can be transformed into circular polarization by shifting the phase of a linear component by  $90^\circ$ . Transmitted circular polarization can be defined in terms of horizontal and vertical polarizations, where circular polarization circulation view is from an observer located at the transmitter [5]:

The inverse transform for transmitted



linear in terms of transmitted circular is

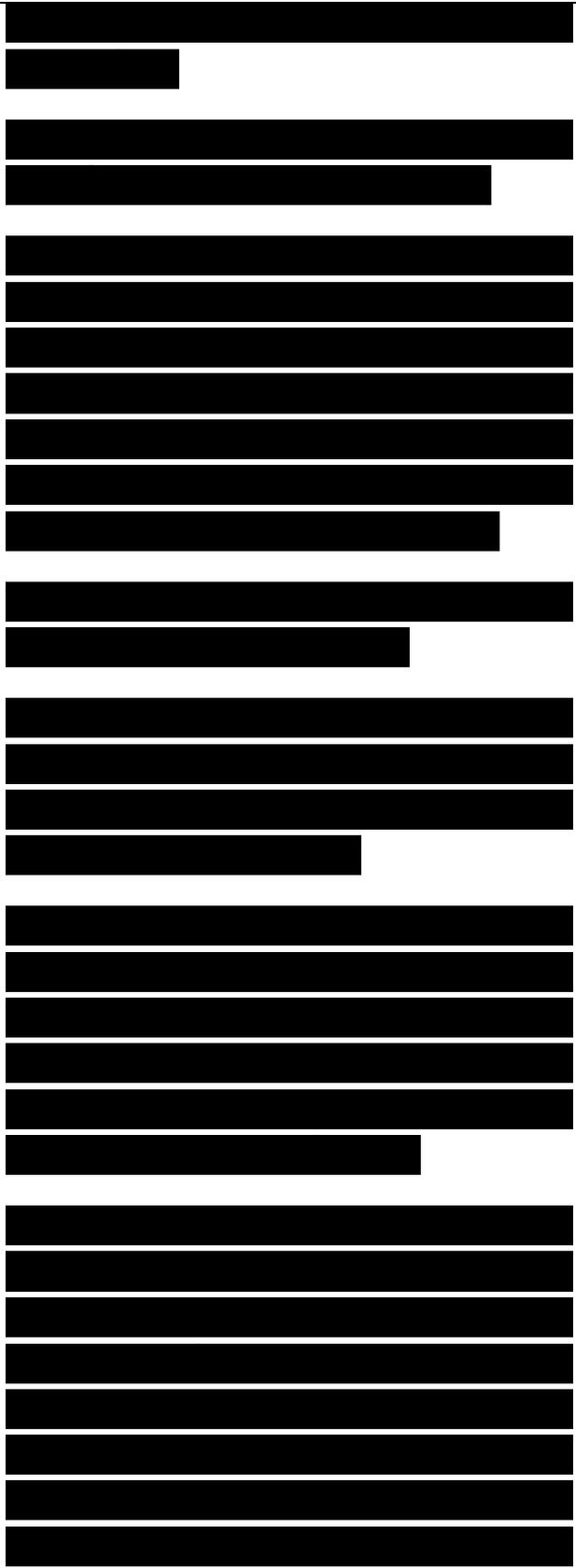
as we may verify by taking the matrix inverse of (3.20).

Received polarization can also be defined in a similar manner, except now the lc and rc definitions change because the viewer is now looking in the direction of propagation, which is from the target toward the receiver, and the radar system has defined lc and rc as looking away. Therefore, which is seen to be the complex conjugate of the transmitted case (3.20).

Figure 3.4. Right circular polarization for transmitting and receiving directions. RC is defined as clockwise rotation of E when viewed in direction of propagation.

The circular polarization PSM contains no more information than the linear PSM. If one has computed or measured a linear PSM, the corresponding circular PSM can be obtained by using (3.20)-(3.22) to obtain [5]

A characteristic feature of circular polarization is that single-bounce scattering changes the polarization from lc to rc or rc to lc. For linear polarization singlebounce specular scattering, the scattered energy has the same polarization as the incident polarization. This occurs due to the scattered field having a 180° phase shift from the incident field; that is, in the opposite



direction (reflection coefficient  $R = -1$ ).

### 3.3 FUNDAMENTAL SCATTERING MECHANISMS

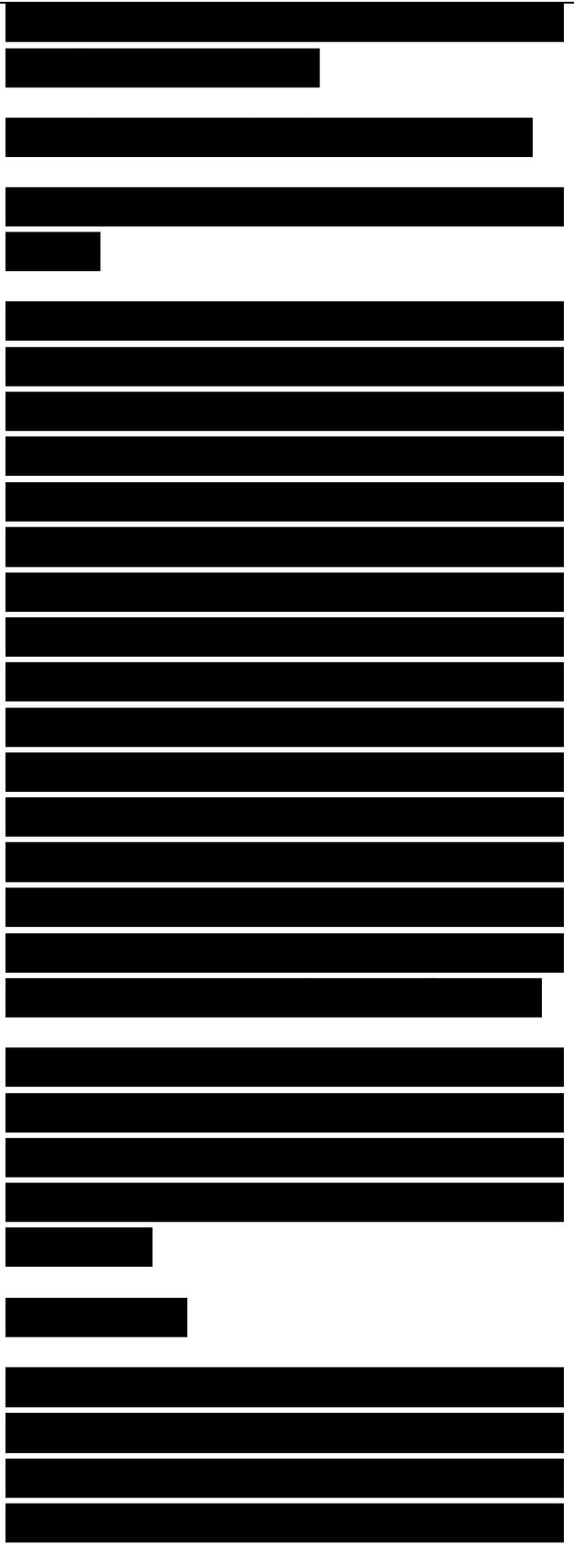
#### 3.3.1 Electromagnetic Wave Fundamentals

An electromagnetic wave is vector in nature and composed of both electric  $E$  and magnetic  $H$  fields, which are able to propagate by themselves. As we shall see later, a time-changing  $E$  field is the source for  $H$  and a time-changing  $H$  is the source for  $E$ . Therefore once launched, an EM wave is able to propagate on its own. EM waves propagate in free space as well inside material media. All EM waves decay in magnitude as they propagate away from their launching source due to spherical spreading, unless anomalous propagation occurs, such as in ducted propagation.

The three most fundamental characteristics of an EM wave are related. The wavelength (spatial variation) times the frequency (temporal variation) is equal to the velocity of propagation:

$$\lambda f = v \quad (3.24)$$

Wavelength  $\lambda$  represents the spatial distance over which the field quantities make a complete cycle; that is, change in value from zero to a positive peak, back through zero to a negative peak,

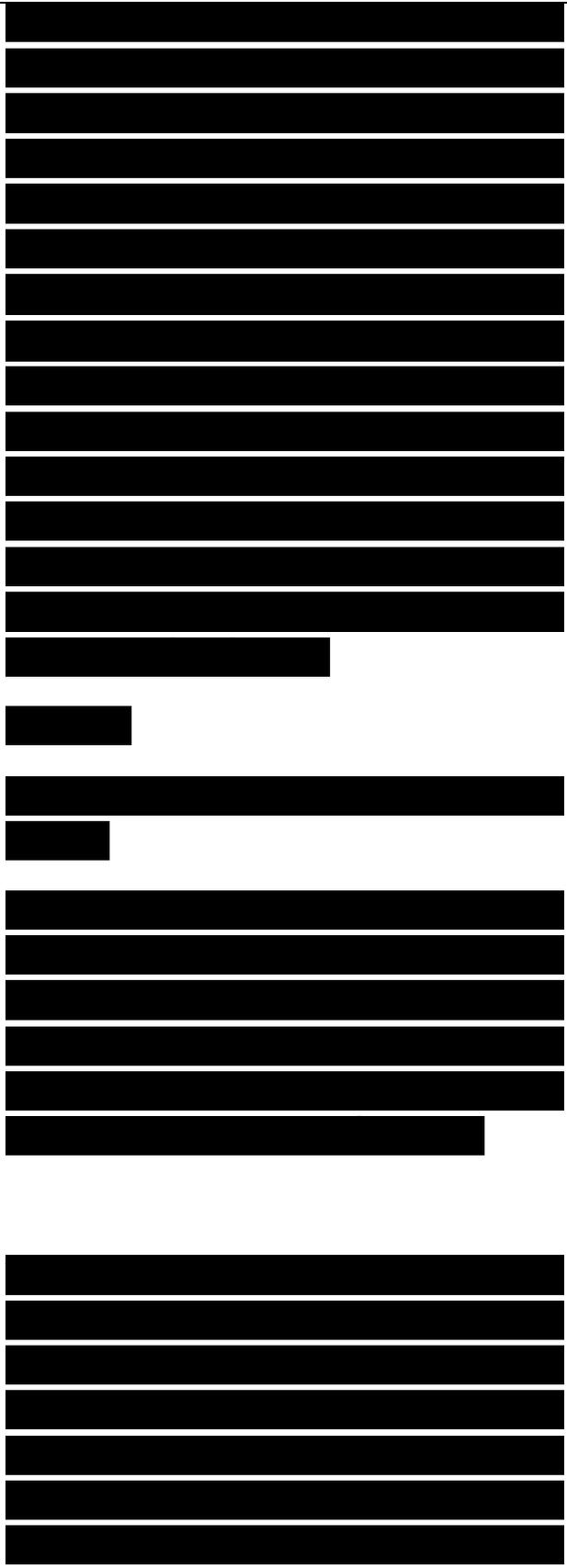


and back to zero, measured in distance, Figure 3.5. The direction of propagation of an EM wave is specified by the wave vector  $k$ , which has a magnitude inversely related to wavelength,  $k = 2\pi/\lambda$ . Frequency represents the number of cycles per second for the wave, measured in Hertz. Radian frequency  $\omega$  is  $2\pi f$ . Alternately the reciprocal of frequency,  $T = 1/f$ , represents the time required for a wave to make a complete cycle. The maximum velocity of an EM wave occurs in a vacuum and is the speed of light, approximately  $3 \times 10^8$  m/s. Wavelength scales can be very long, such as  $5 \times 10^6$  m  $\sim$  3107 mi for 60 Hz

Circular Elliptical  
Figure 3.5. Wave nature of an electromagnetic field.

radiation, to very short such as  $10^{-7}$  m for light. For typical radar applications Table 3.1 shows the range of wavelength and frequency values usually of interest. Although this is certainly only a small part of the EM spectrum, it is nevertheless a broad range of values.

Sources for E and H fields are charges and currents. Near sources, the field lines originate on local charges; that is, the field lines are conservative. As the fields propagate away from sources, they can no longer remain attached to the source charges. Now they must close back on themselves in a solenoidal fashion. This is the case for a free-space

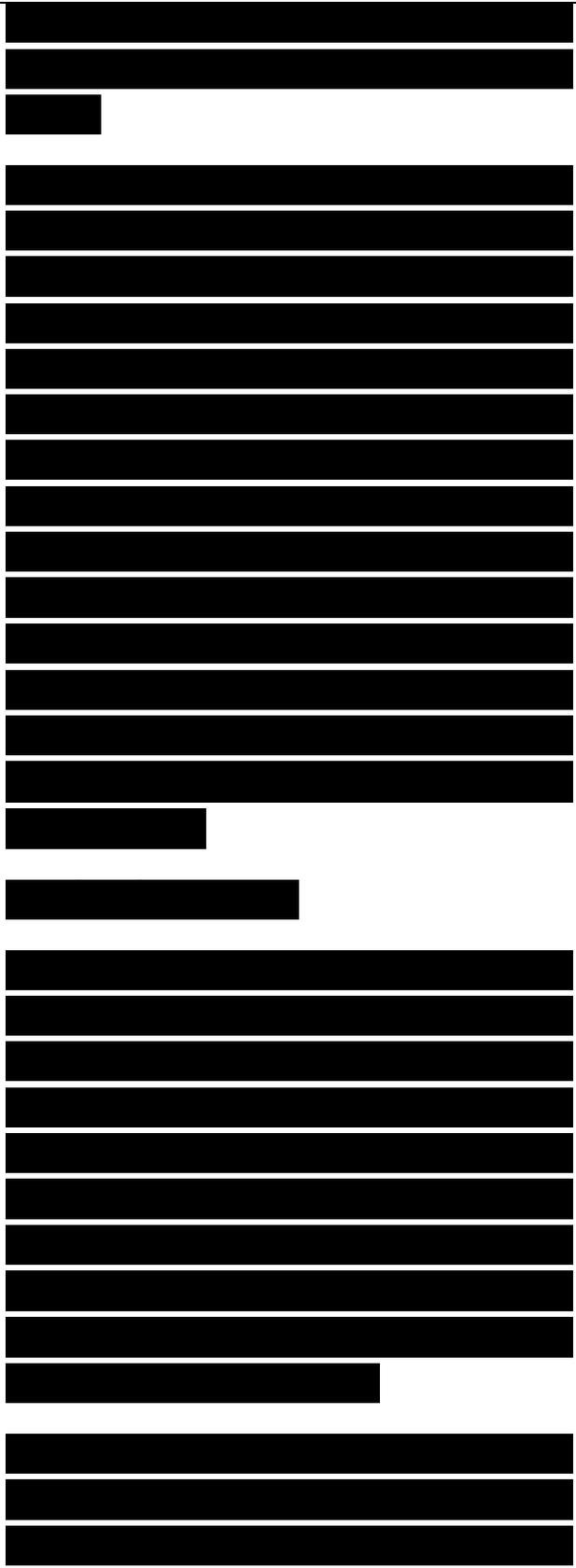


EM wave.

The direction of E and H must be perpendicular to k. Therefore E and H must reside in a plane perpendicular to k. The directions of E and H are still somewhat arbitrary. The specific direction of E is called the polarization of the wave. It may be linear or circular; that is, it rotates as the wave propagates. For linear polarization the usual directions are horizontal or vertical if we are doing experimental work, or for theoretical work, we refer to a spherical coordinate system, using the polar angle  $\theta$  and azimuth angle  $\phi$  vector directions, Figure 3.6.

Band Frequency Wavelength

In free space, the E and H fields are perpendicular to each other and to the direction of propagation k, Figure 3.5. The electric field E has units of (volts / meter) whereas the magnetic field //has units of (amperes/meter). The propagation vector k points in the direction of travel of the wave and has a scalar magnitude related to the reciprocal of wavelength,  $k = 2\pi/\lambda$ , m<sup>-1</sup>. In free space the E and H fields are in phase; that is, when E peaks so does H. An EM wave represents the transport of energy. This is specified in terms of power flux density, watts/meter<sup>2</sup>, and is vector in nature because a spatial



direction is involved. This is the Poynting vector defined as

### B) Analytical Spherical Coordinate System

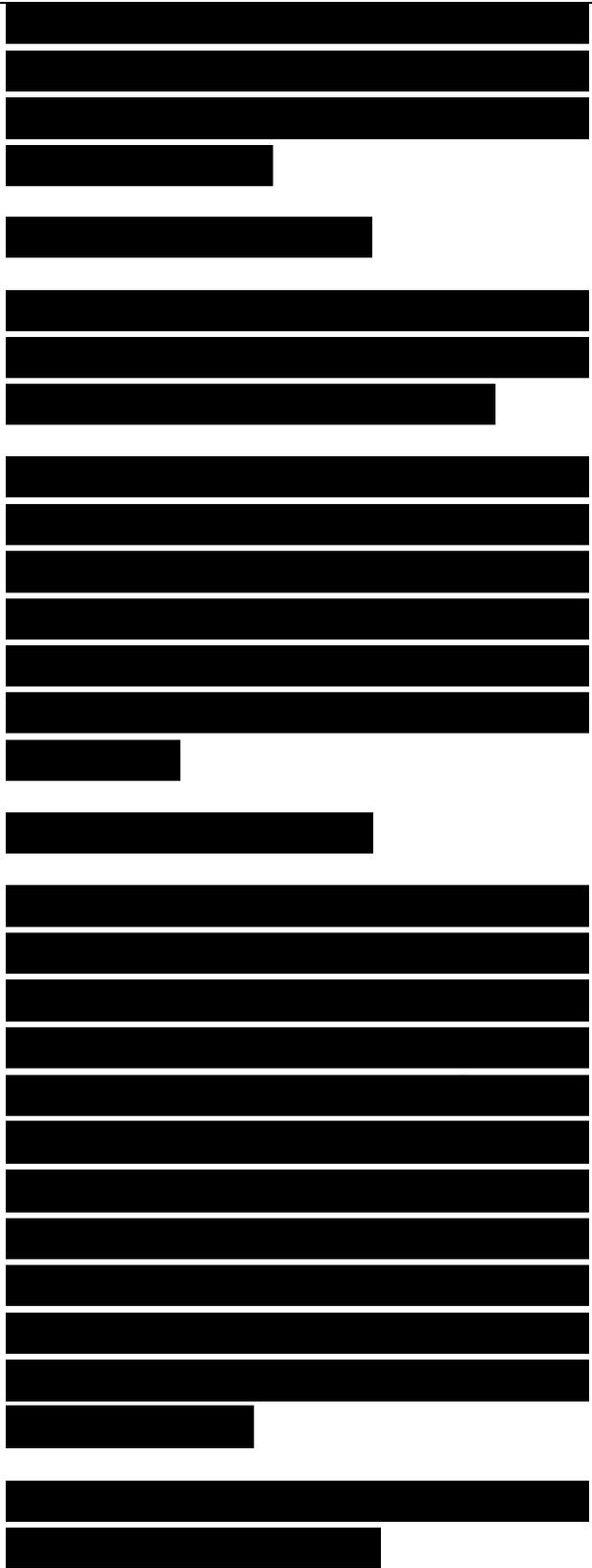
Figure 3.6. Typical linear polarizations for experimental and analytical work.

E and H fields also represent energy storage. Energy is split equally between the E and H fields. The energy density is given in terms of the E and H field quantities and parameters that characterize the material ability to store energy:

$$U = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \quad (3.26)$$

Permittivity,  $\epsilon$ , characterizes a materials ability to store electrical energy. It is related to capacitance and has units of farads per meter. The free-space value, denoted by the subscript zero, is approximately  $8.85 \times 10^{-12}$  f/m. Permeability,  $\mu$ , characterizes a materials ability to store magnetic energy. It is related to inductance and has units of henrys per meter. Its free space value is defined exactly as  $4\pi \times 10^{-7}$  h/m.

The velocity of an EM wave is inversely related to energy storage,  
 $v = \frac{1}{\sqrt{\epsilon \mu}} = c \quad (3.27)$



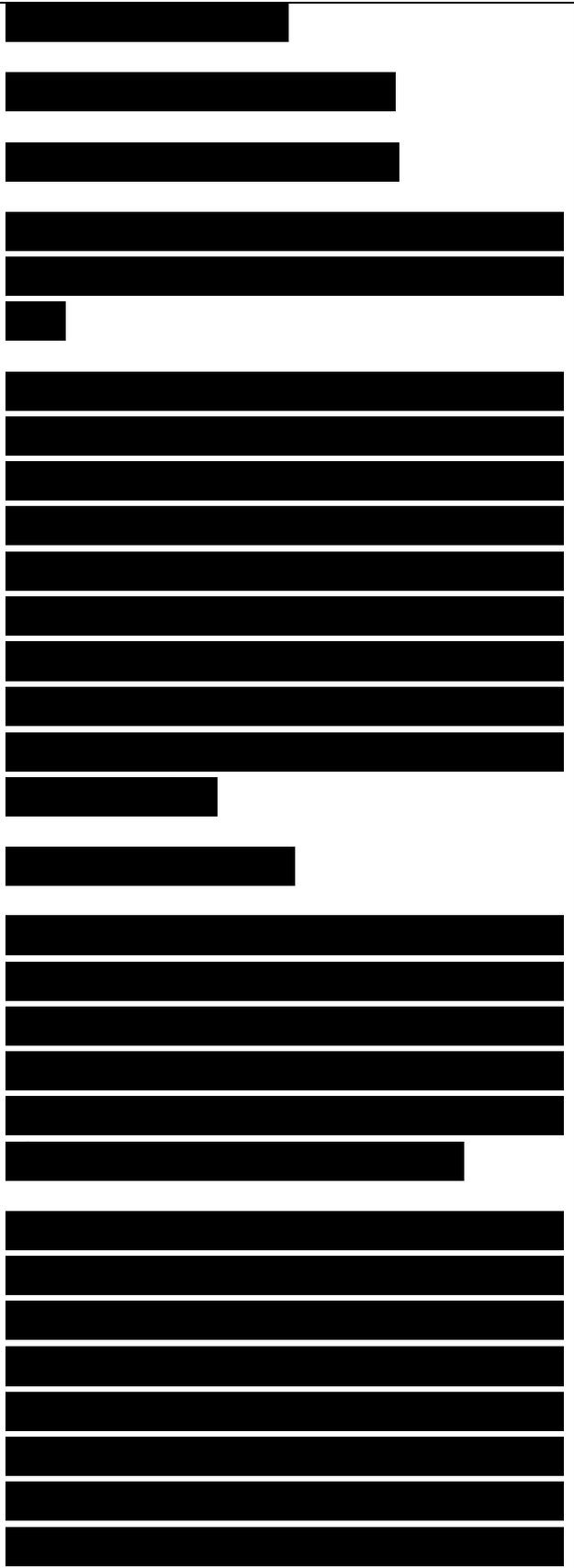
which for free space has the value  
 $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \text{ m/s} \quad (3.28)$

The speed of light in a vacuum represents the least storage of energy.

Actual values for E and H fields, although sometimes specified as microvolts or microamps per meter, are usually not of interest. They always decay with distance away from a source due to spherical spreading. However, the ratio of E to H is of interest, and it is called the wave impedance. In free space it is  
 $\approx 120 \pi \approx 377 \text{ } \Omega \quad (3.29)$

Although equal energy is contained in E and H, their numeric values differ by the value of the wave impedance. When a wave is near a conducting surface where the tangential E must become small or zero, the wave impedance becomes small.

In a material medium the character of an EM wave differs from free space due to varying amounts of energy storage in E and H fields. Because all materials store at least some electrical or magnetic energy, the wave velocity is always less than free space. Then, depending on specific values of  $\epsilon$  and  $\mu$  the wave impedance is no longer 377 (unless  $\epsilon = \epsilon_0$  and  $\mu = \mu_0$ )

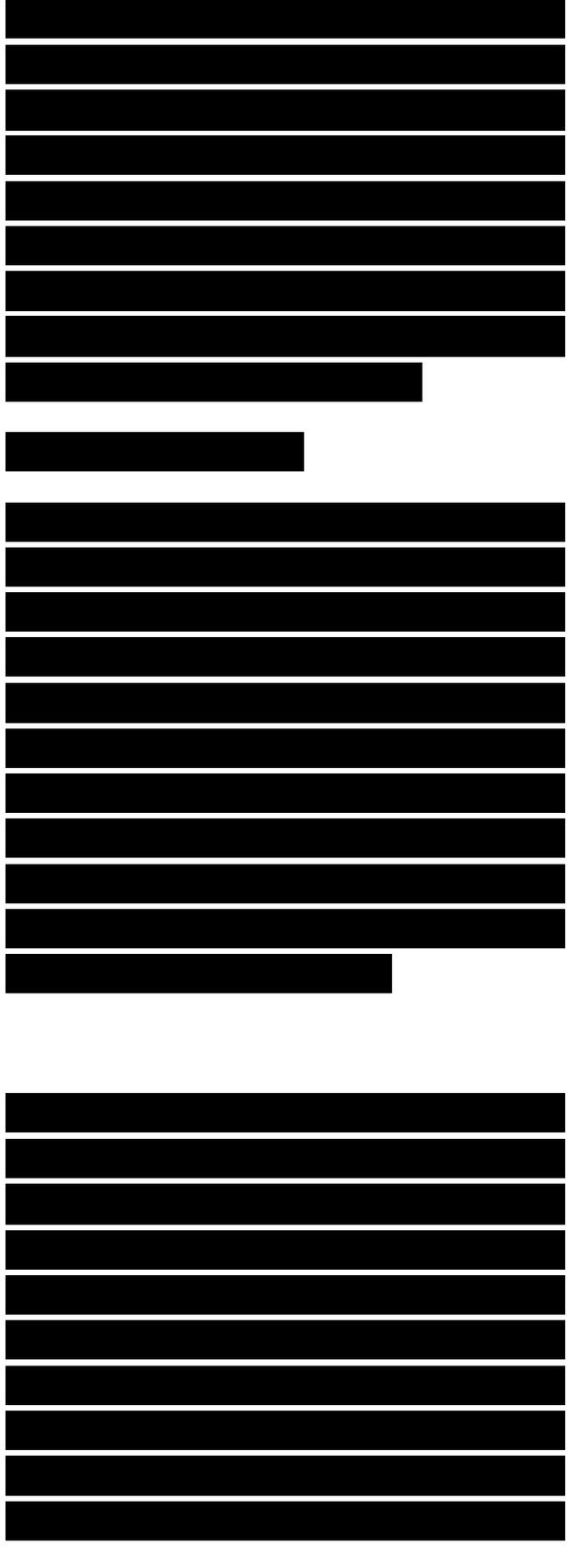


//), and there may be a phase difference between E and H; that is, they do not peak at the same time. A wave propagating in a conducting medium (but not a perfect conductor) has  $\epsilon'' \ll \epsilon'$ , and E lags behind H typically by  $45^\circ$  due to storage of electrical energy.

### 3.3.2 The Scattering Process

The scattering process can be characterized in two ways. The first is to think of an EM wave as a billiard ball that reflects or bounces off surfaces often in a specular manner; that is, angle of incidence = angle of reflection. This view does not examine the details of the interaction of the wave with a surface. The second approach is to consider the details of the interaction, which involve induced charges and currents and the fields that they reradiate.

When an EM wave propagating in free space impinges on a material object characterized by  $\epsilon$  and  $\mu$ , not free-space values, energy is reflected, transmitted, or absorbed, Figure 3.7. Because radar cross section is concerned principally with scattering from conducting surfaces, let us specialize our scattering process arguments for this case. A perfect electric conductor (PEC) is characterized by  $\epsilon_r = \epsilon' - j\epsilon''/\omega\epsilon_0 \rightarrow \infty$  as the conductivity  $\sigma$ , the reciprocal of



resistivity, is infinite. This would suggest that a PEC could store an infinite amount electrical energy, a physical impossibility. Thus the electric field must be zero in a PEC. Another view of a conductor is that its electrons are free to move instantly in response to an electric field. However, because these electrons represent a charge density, they create their own electric field, which we call the scattered field. These electrons can move only so long as the total electric field is not zero. The field created by

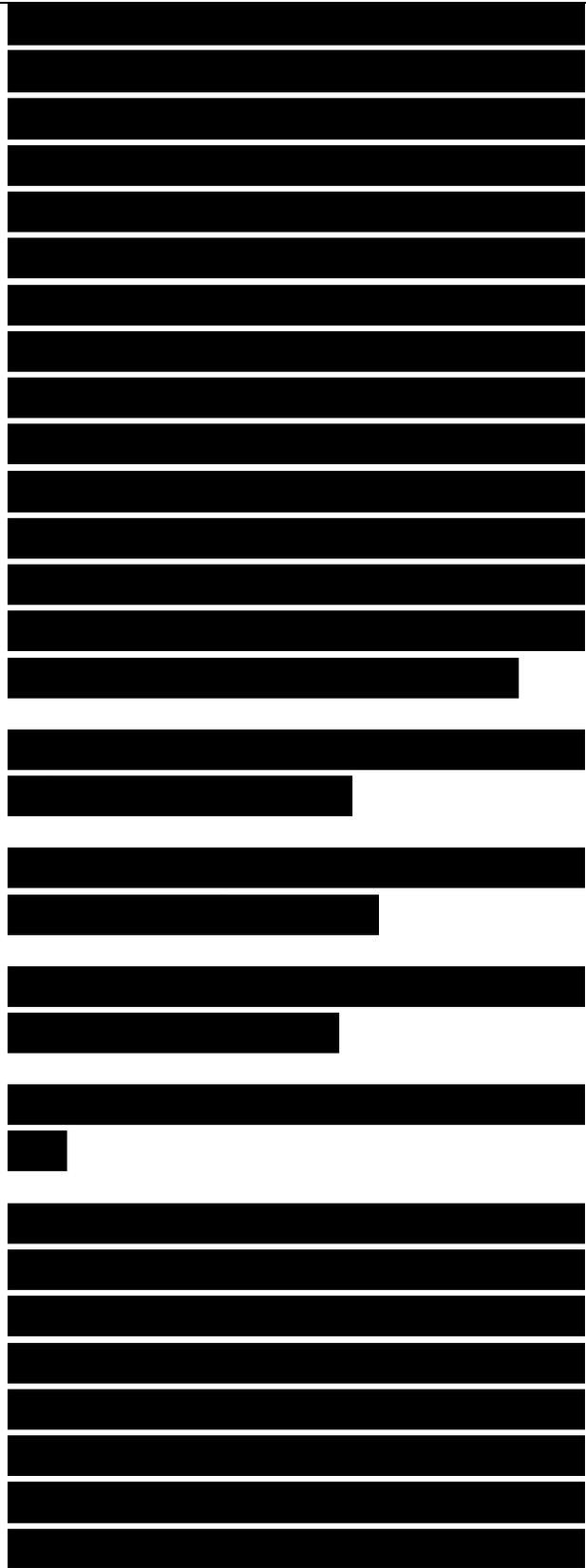
Incident Field Characterized by Direction and Polarization

Scattering Body on which are Induced Currents and Charges:

Envelope of Scattered Field due to Induced Sources on Scatterer

Figure 3.7. Basic electromagnetic scattering process.

these electrons is in the opposite direction to the applied field. Therefore, when the scattered field is equal and opposite to the incident field, the total field on the conductor is zero, and a force is no longer acting to move the electrons. This is the notion that a PEC surface has a boundary condition of zero tangential electric field.



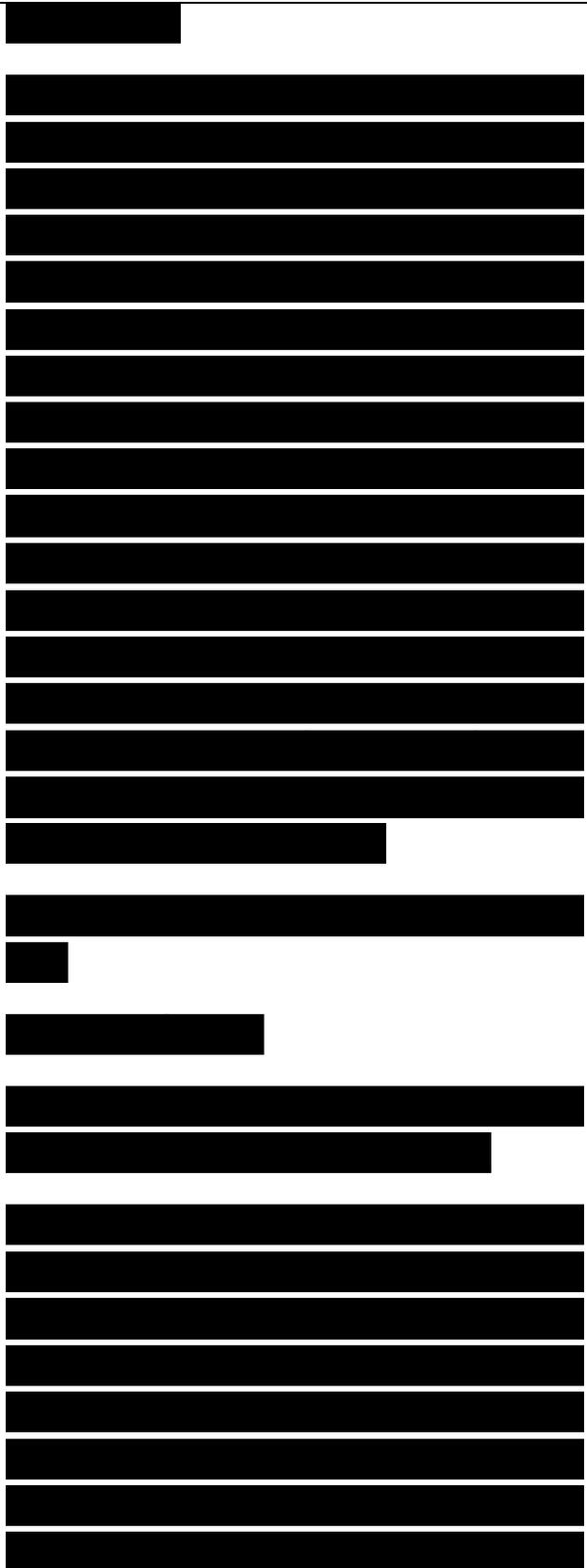
This instantaneous equilibrium does not last. The incident wave is a time-changing field. The free electrons move in response to the changing incident field to always keep the total tangential surface field zero. With Figure 3.8 showing the background geometry and field computation, a time sequence is shown in Figure 3.9 for a IX square plate geometry illuminated perpendicular to the plate with  $\epsilon'nc$  along the x direction. Four time values are shown,  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  phase. (The remainder of the time sequence from  $90^\circ$  to  $360^\circ$  is a repeat of the  $0^\circ$  to  $90^\circ$  quarter, but with differing signs.) At  $0^\circ$  time phase, the incident E field is a maximum at

Illumination direction for Figures 3.9 and 3.10

Illumination direction

Figure 3.8. Geometry and field computation plane for Figures 3.9 to 3.13.

the plate, and the scattered field by the plate is the opposite direction to make the total tangential field zero. Later in time the incident wave peak passes beyond the plate and the plate-scattered field begins to propagate out and away from the plate, as seen in Figure 3.9(b) through 3.9(d). At time phase of  $90^\circ$ , the incident wave has a null at the plate.

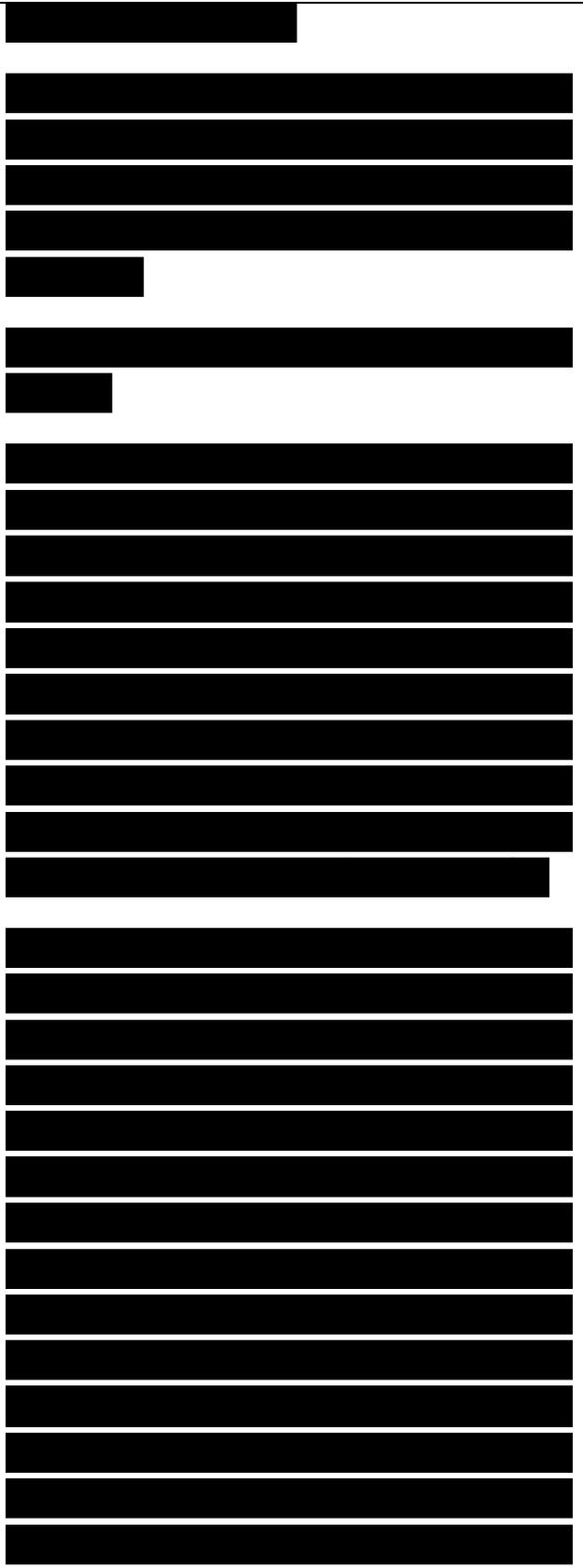


The time-varying incident field causes a time-varying charge separation to occur on the conductor which represents a current flow. These charges and currents

(a) E scattered,  $t=0$  deg (b) E scattered,  $t=30$  deg

represent the sources for the scattered field. As the charges move, the attached field lines move with the charge. Field lines more than  $k/2$  away from the surface cannot keep up with the charge movement due to the finite speed of light. The more distant field lines begin to close back on themselves and propagate on their own away from the source charges; that is, an EM wave is launched and becomes a self-propagating entity.

In the Fresnel or near zone, the E field lines end on surface charges, and the fields are mostly conservative in nature. In the Fraunhofer or far field, the E fields completely close back on themselves, the field is solenoidal. An example of a near- to farfield transition for scattered field contour levels (but not vector direction) is shown in Figure 3.10 for a  $2\lambda$  plate illuminated perpendicular to the plate as shown in the geometry illustration of Figure 3.8. The scattered field is symmetric about the plate; that is, the reflected and forward waves are the same as is required by symmetry. The forward-scattered wave is out of phase with the incident field so, when



the two are added, a shadow is formed behind the plate. The two major lobes are the forward and reflected lobes in addition to four minor lobes at  $\pm 45^\circ$ .

A very convenient description for E and H fields is to decompose the total field into an incident part due to sources that are far away and a scattered part due to the charges and currents induced on a scattering body.

$$P_{total} = P_{incident} + P_{scattered}$$

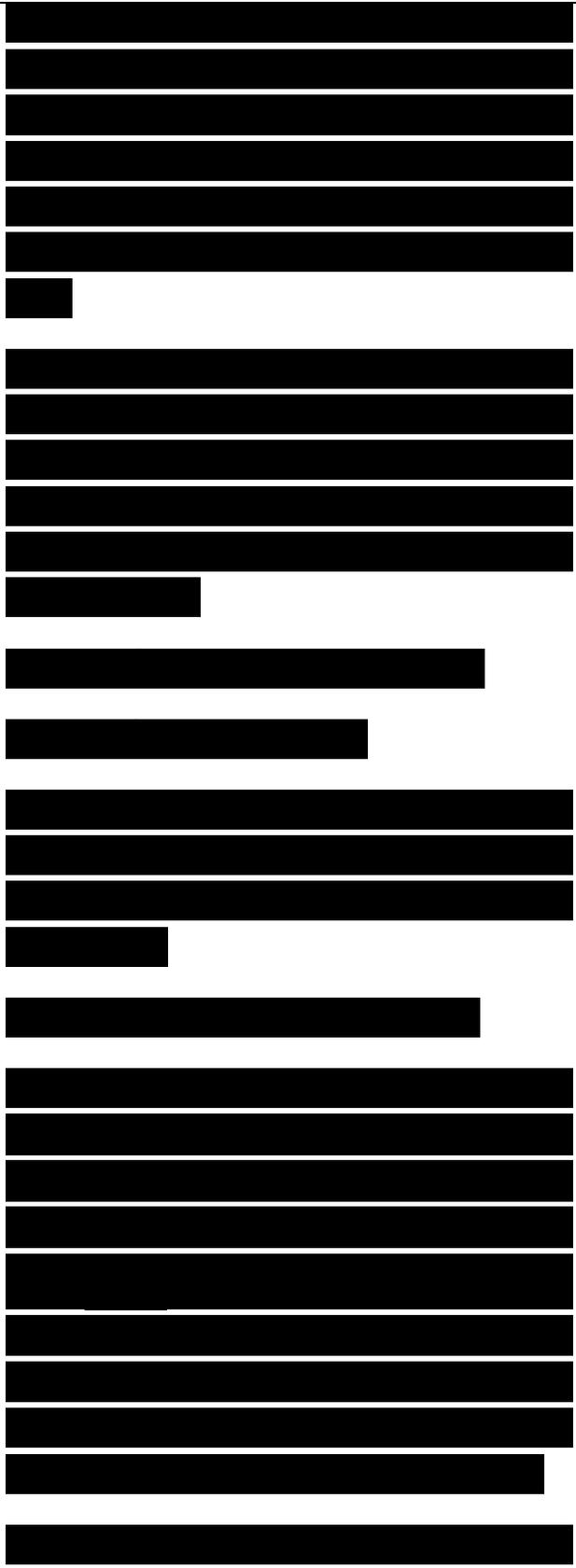
$$j_{total} = j_{incident} + j_{scattered}$$

The incident field, which is spherical with its  $1/R$  spatial decay, is often taken as a plane wave in the target vicinity; that is,

$$E_{incident} = u \exp(j(kR - \omega t)) \quad (3.31)$$

which represents an incident plane wave with polarization direction  $u$ , direction of propagation  $k$ , and frequency  $\omega$ . Because  $v = c$ , the radian frequency  $\omega = 2\pi f$  and wave number  $k = 2\pi/\lambda$  are related,  $\omega/k = c$ . An example of an incident plane wave magnitude in the  $x-z$  plane (Fig. 3.8) is shown in Figure 3.11, for  $\omega t = 0$ , traveling toward the origin at  $45^\circ$  with respect to the  $x$  axis (not very exciting!).

The field scattered in the  $x,z$  plane (Fig.



3.8), that is, radiated by induced charges and currents, by a  $2k$  plate illuminated at  $45^\circ$  is shown in Figure 3.12, for  $wt = 0$ . We see two principal scattered-field directions, one reflected mostly in the specular direction (angle of incidence = angle of reflection) and one in the forward direction. This latter component is out of phase with the incident field so that it subtracts from the incident field to form a shadow behind the plate.

Distance Along Plate (m)

Figure 3.10. Scattered field from  $2\lambda$  plate excited normal to plate.

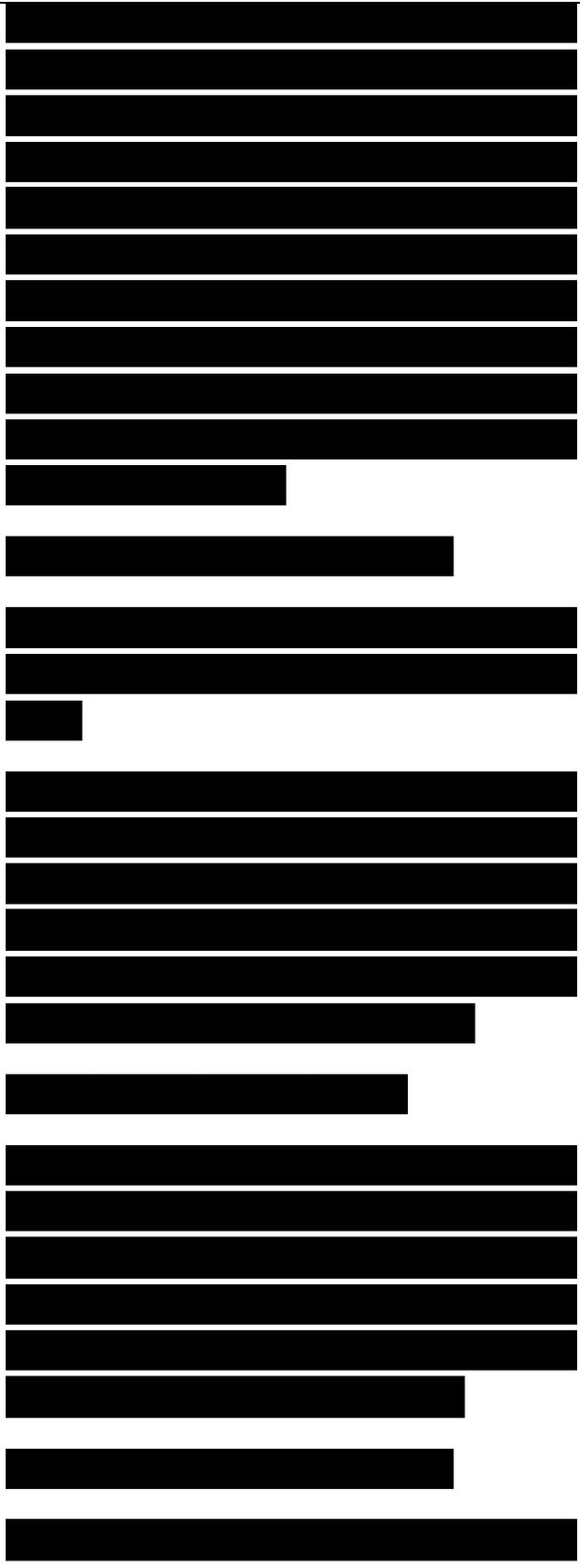
The total field is the sum of the incident and scattered components. This is shown in Figure 3.13, for  $\cot = 0$ , where we can clearly see the shadow behind the plate and the interference pattern of the specular scattered field with the incident wave.

### 3.4 SCATTERING REGIMES

Three regimes characterize RCS scattering, depending on the ratio of wavelength  $X$  to body size  $L$ ,  $\sqrt{L}$  or inversely,  $kL$ . The three regimes are the Rayleigh region, the resonant region, and the optics region corresponding to  $X > L$ ,

Distance Along Plate (m)

Figure 3.10. Scattered field from  $2\lambda$  plate excited normal to plate.



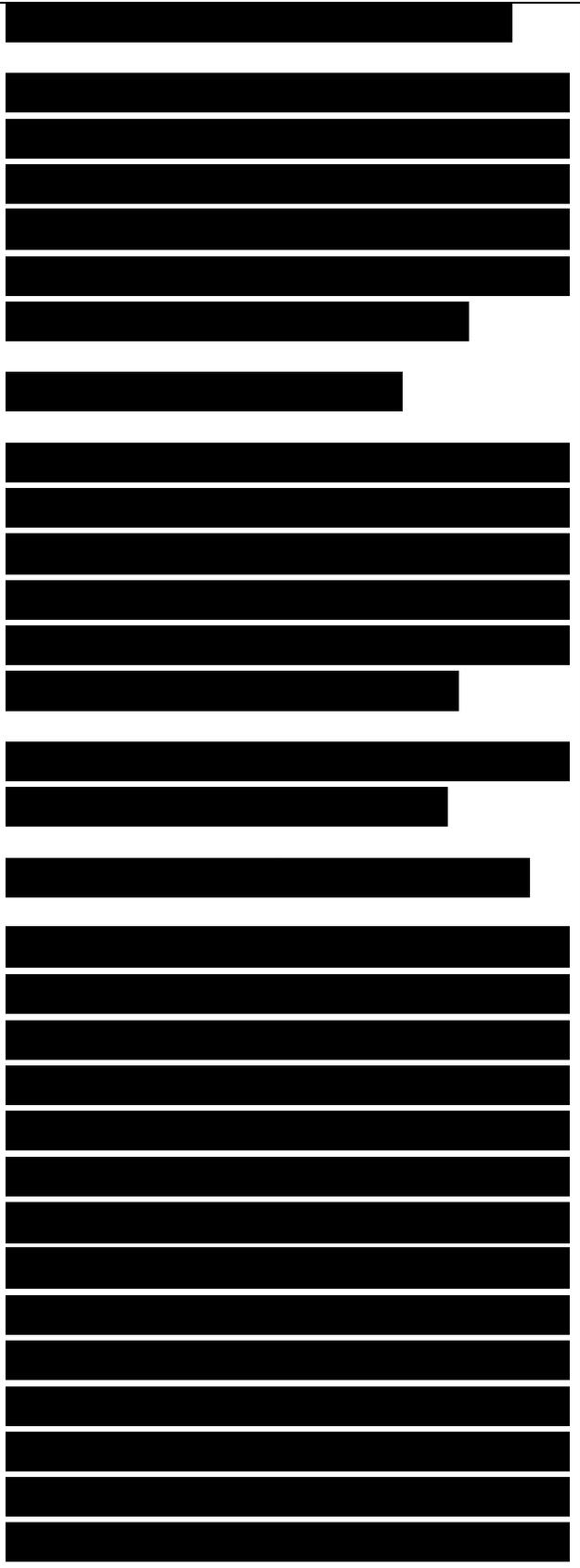
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-3.00 -2.00 -1.00 0.00 1.00 2.00 3.00  
Distance Along Plate (m)

Figure 3.11. Incident plane wave at  $45^\circ$ .  $X. a L$ , and  $X < L$ . The classic illustration of cross section over these three regions is that of a sphere as shown in Figure 3.14, where  $\sigma_r$  has been normalized to the projected area of the sphere,  $\pi a^2$ , plotted as a function of sphere circumference normalized to wavelength,  $ka = 2\pi a/X$ . When the wavelength is much greater than the sphere circumference, its cross section is proportional to  $\pi a^2(ka)^4$ , which shows us that, although  $\sigma_r$  is small, it increases as the fourth power of frequency and sixth power of radius. When the circumference is between 1 and 10 wavelengths the cross section exhibits an oscillatory behavior due to the



interference of the front- face optics like return and the creeping wave that propagates around the sphere. This is known as the resonant region. When the circumference is large compared to a wavelength, the oscillatory behavior dies out as the creeping wave mechanism

Distance Along Plate (m)

Figure 3.12. Scattered field from 2\ plate illuminated by a plane wave at 45°.

disappears, and we are left with only the front-face optics reflection, which for a doubly curved surface is  $a = na^2$ , the projected area of the sphere. This is the optics region.

The dominant scattering mechanism in the Rayleigh region is induced dipole moment scattering. In the resonant region, optics and surface wave mechanisms dominate the scattering, and in the optics region, surface wave effects are minimal.

### 3.4.1 Low-Frequency Scattering

When the incident wavelength is much greater than the body size, the scattering is called Rayleigh scattering. This is named after Lord Rayleigh's analysis of why

Distance Along Plate (m)

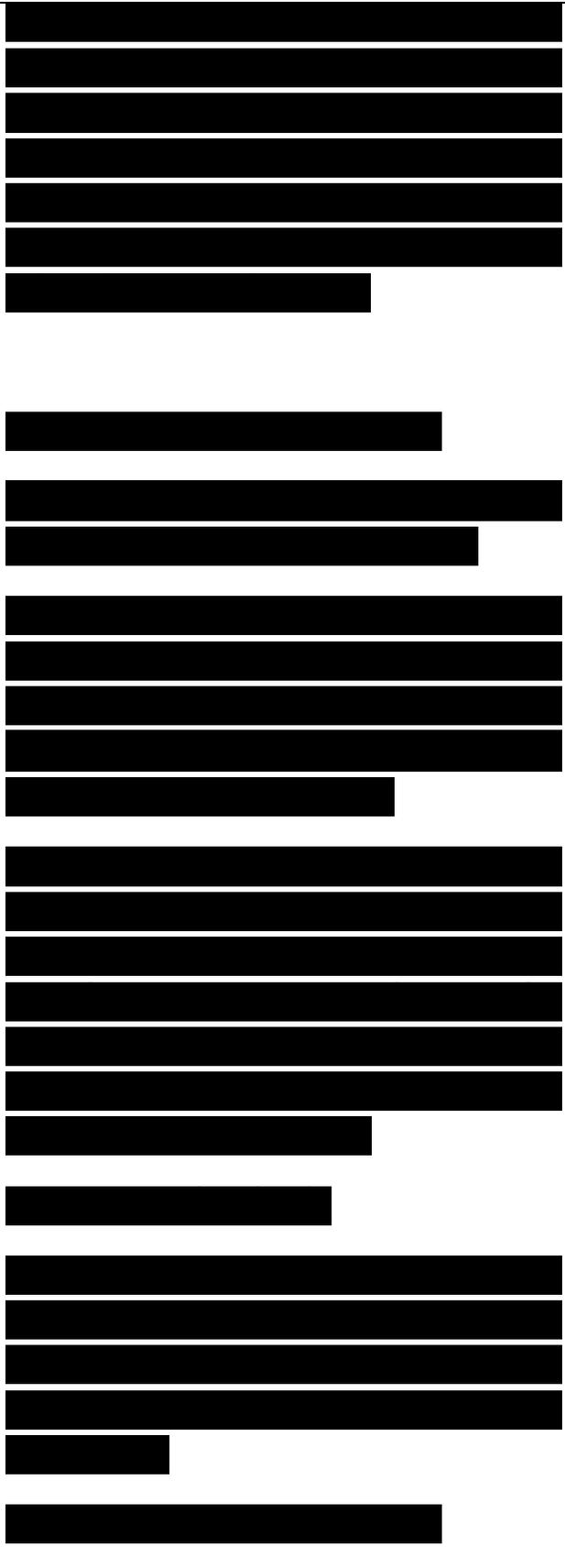


Figure 3.13. Total field from 2k plate illuminated by a plane wave at  $45^\circ$ .

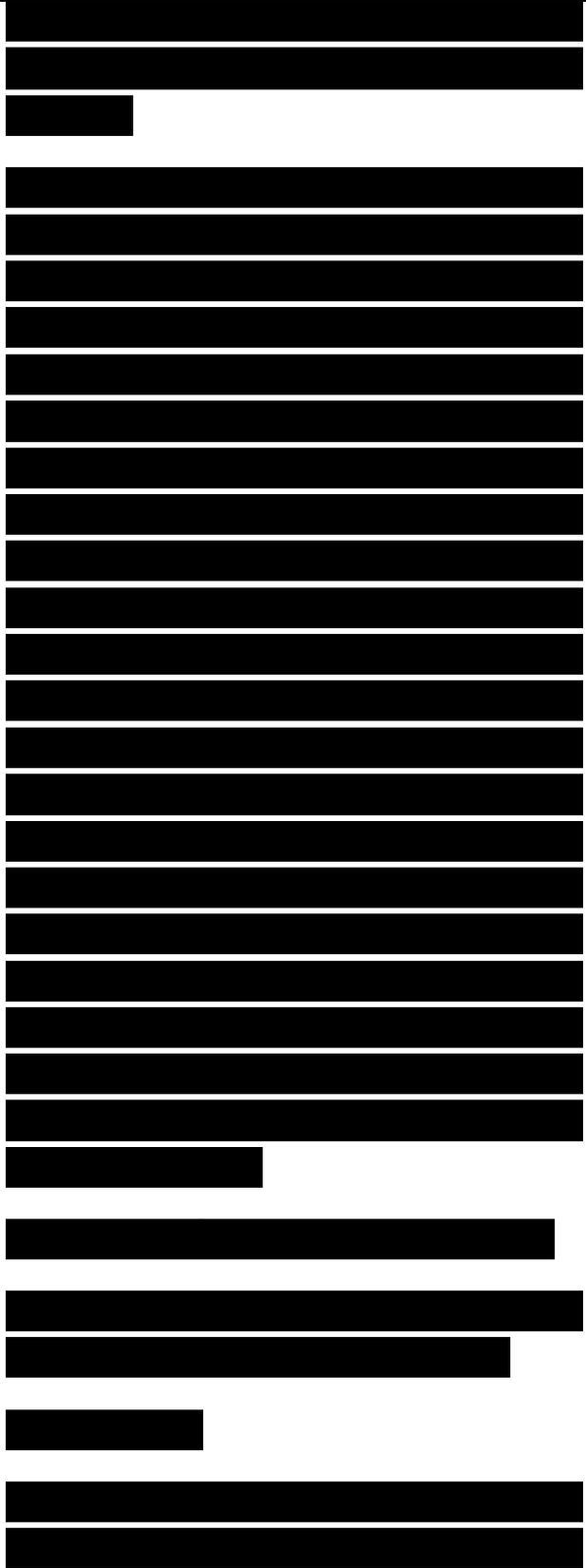
the sky is blue: the shorter blue wavelengths are more strongly scattered than the longer red wavelengths. In the low-frequency case, there is essentially little phase variation of the incident wave over the spatial extent of the scattering body: each part of the body “sees” the same incident field at each instant of time. This situation is equivalent to a static field problem, except that now the incident field is changing in time. For linear polarization, the vector direction of the incident field does not change with time, as shown in Figure 3.15. For circular incident polarization, the situation can be understood by decomposing the incident polarization into two orthogonal linear polarizations, one shifted in phase by  $90^\circ$  with respect to the other. This quasistatic field builds up opposite charges at the ends of the body; in effect, a dipole

### Sphere Circumference in Wavelengths

Figure 3.14. Radar cross section of a metallic sphere over the three scattering regimes.

Scattering Body

Figure 3.15. In the low-frequency region there is little variation in either the amplitude or phase of the incident field



over the body length.  
moment is induced by the incident field. The strength of this induced dipole is a function of the size and orientation of the body relative to the vector direction of the incident field. Dipole moments are defined as charge density times separation distance. For example, when the applied field is perpendicular to a long body, the induced dipole moment is less than the moment induced when the applied field is parallel to the body axis, as suggested in Figure 3.16.

The salient characteristic of Rayleigh scattering is that cross section is proportional to the fourth power of the frequency or wave number:  
 $\sigma \propto \omega^4$  or  $\sigma \propto k^4$  (3.32)

This behavior can be explained qualitatively by reference to the expression for scattered field, (3.100), where  $E_s$  is a  $\frac{1}{r}$ . There  $J$  is a current density related to charge by  $J = dq/dt = \omega q$ . Therefore  $E_s \propto \frac{1}{r} \omega^2 q$  so that the radar cross section, which is proportional to  $(E_s)^2$ , depends on  $\omega^4$ .

Because Rayleigh scattering is essentially a static field problem, all the analytical procedures for electrostatics can be invoked. These include the integral

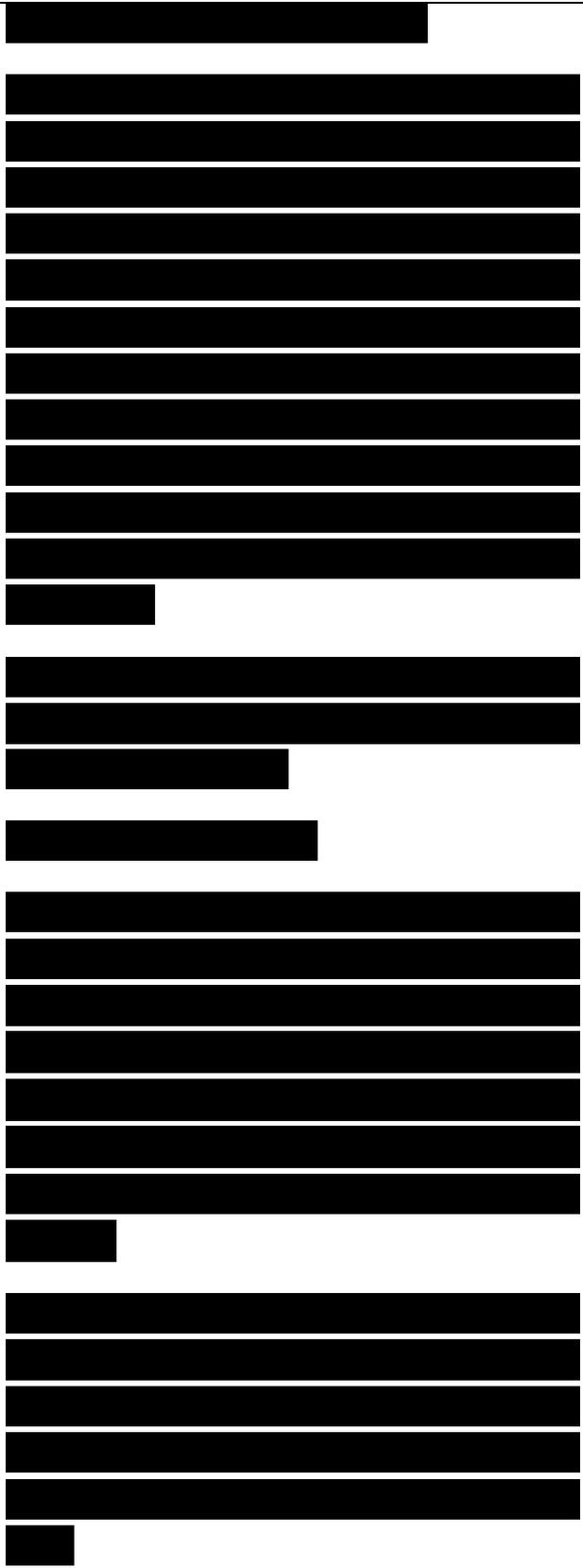


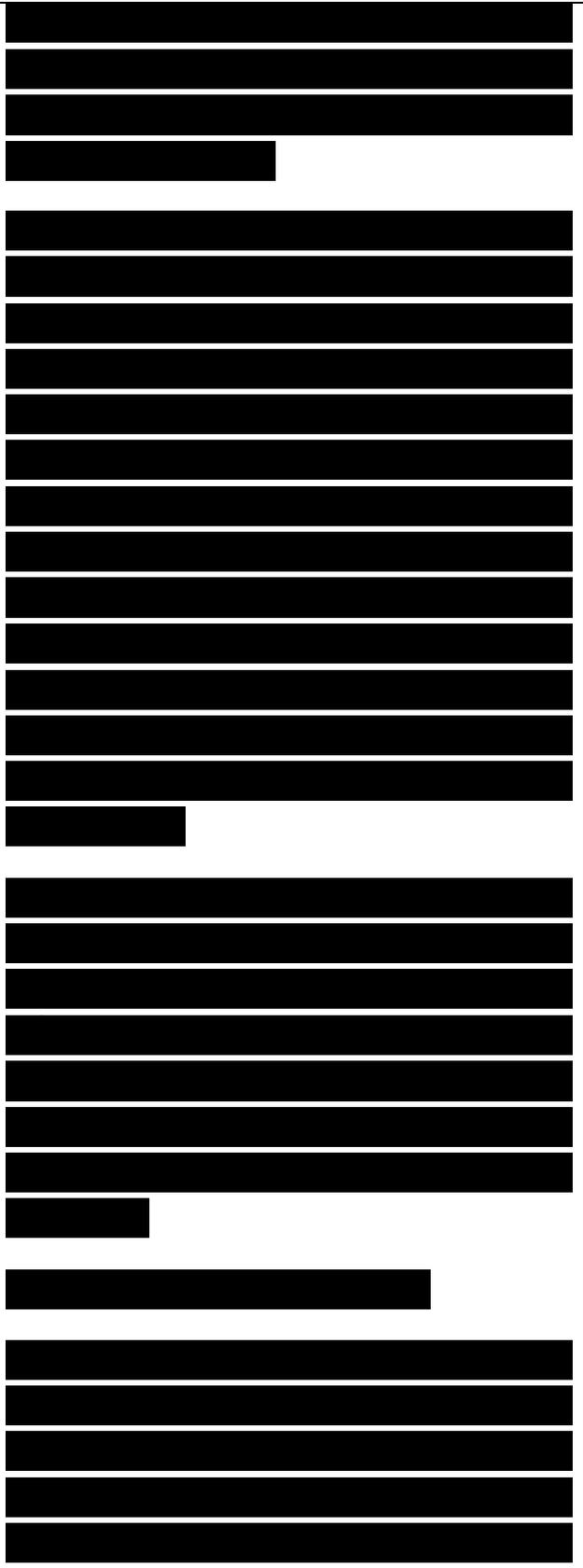
Figure 3.16. Low-frequency induced dipole moments:  $P$  is the induced dipole moment,  $L$  is the body length, and  $d$  is the body diameter.

equation approach (the solution to Poisson's equation) and the dipole and multipole expansions. A scalar approach, rather than a vector approach, is possible because induced charge density is the chief physical mechanism, a scalar quantity. For low-frequency scattering, the entire body participates in the scattering process. Details of the shape are not important, and therefore, only a basic or crude geometric description is required because gross overall shape is more important than detailed shape information.

For most applied problems, the wavelength is usually small compared to the body size so that Rayleigh scattering is of little importance. The low-frequency approach can be used until there is appreciable phase change of the incident wave over the length of the scatterer.

### 3.4.2 Resonant Region Scattering

When the incident wavelength is on the order of the body size, the phase of the incident field changes significantly over the length of the scattering body, Figure 3.17. Although there are no absolute definitions, we typically take the

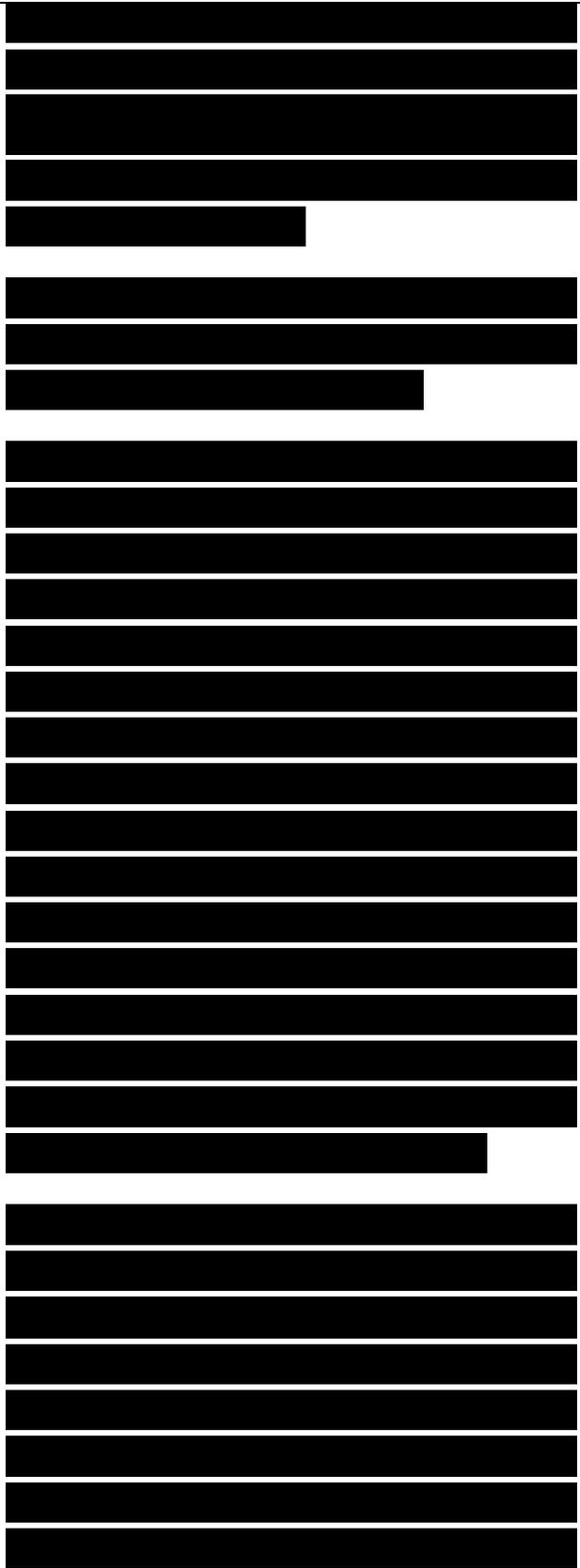


resonant region to be for bodies between 1 and  $10\lambda$  in size,  $1 < L/\lambda < 10$ . In this region we start to have two classes of scattering mechanisms, surface wave effects that

Figure 3.17. In resonance region scattering, the phase of the incident field changes several times along the body length.

are unique to only the resonant region and optical mechanisms. Surface wave effects are nonoptical; that is, scattering does not occur for angle of reflection equal to angle of incidence. The name resonant region is a bit of a misnomer. We seldom deal with high-Q, sharply peaked resonant phenomena. Rather we have physical mechanisms where EM energy stays attached to the body surface. Surface wave types are traveling waves, creeping waves, and edge traveling waves. Surface wave scattering occurs when this surface energy is reflected from some aft body discontinuity or, as is the case for a creeping wave, the energy flows completely around the body.

Surface wave scattering is independent of body size. Cross-section magnitudes are proportional to  $A^2$ ; that is,  $(L^2 A^2)$ . From this relation we see why surface wave effects are important for resonant region body sizes. Surface wave effects are present in the optical region, but the scattering magnitudes are much smaller than optical scattering magnitudes, which most often are proportional to

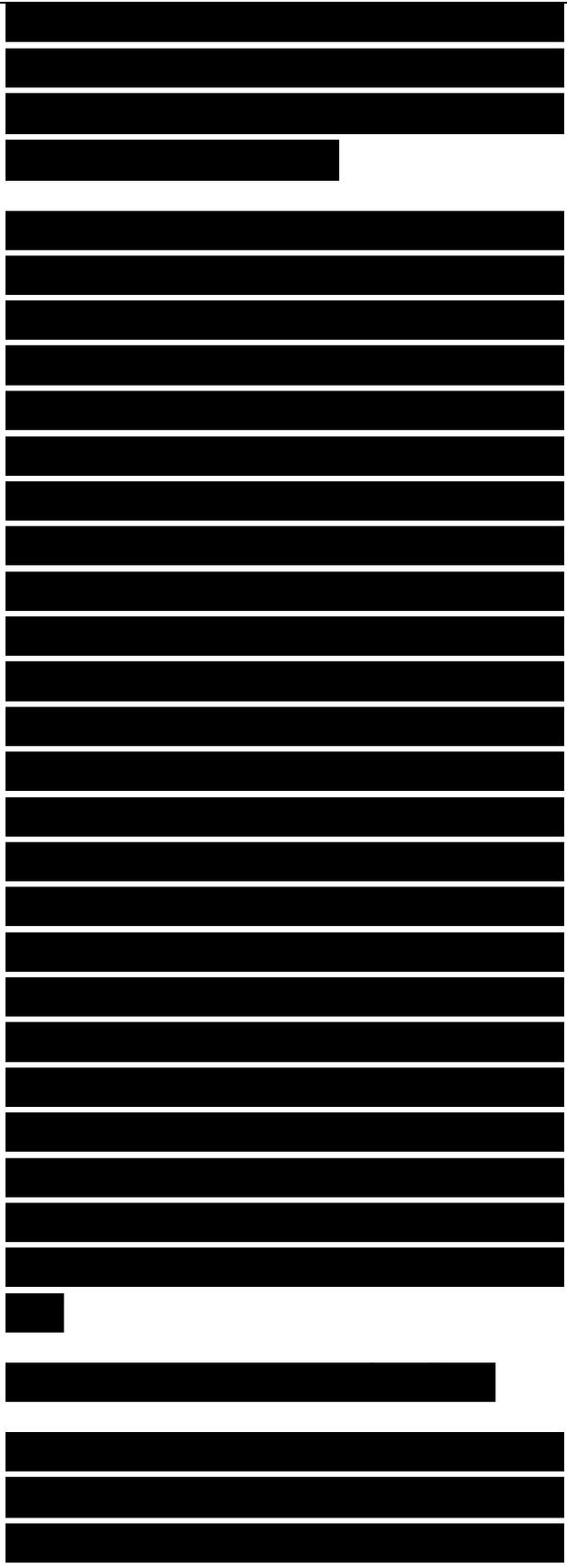


(LA.), (L2A°), (L3, A'1), or (L4, A"2).

In this scattering regime body-body interaction is important: the field at any part of the body is the sum of the incident field plus that scattered by other regions of the body. This collective interaction determines the resultant current density. Overall geometry is important; however, small scale details (relative to wavelength) are not. In this regime an exact solution of Maxwell's equations is required. Typically the method of moments is used to solve the Stratton-Chu integral form of Maxwell's equations to obtain the induced currents from which the scattered field is obtained. Techniques of solving these integral equations have received great attention in the last 30 years. The availability of large high-speed computers has enabled the implementation of integral equation matrix methods. The advent of parallel computers will open this capability considerably.

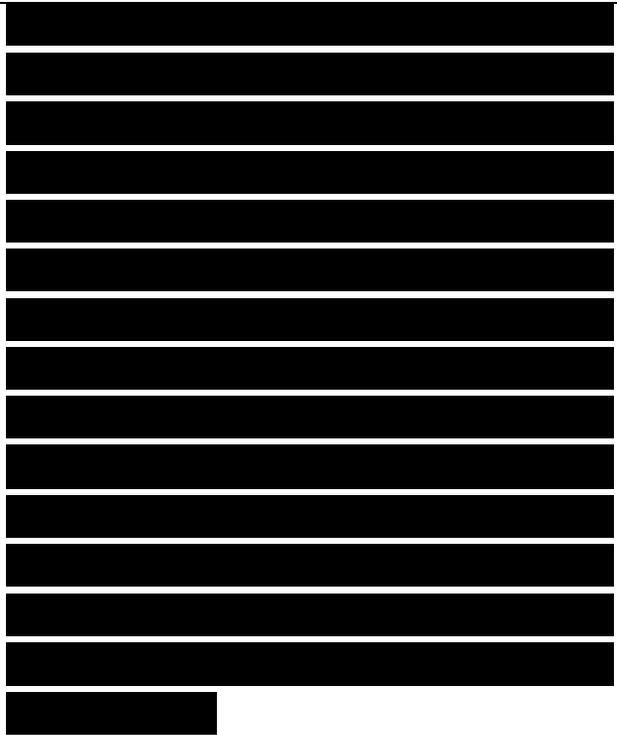
### 3.4.3 High-Frequency Optics Region

When the wavelength becomes much smaller than the body size, typically  $A < 10L$ , a localized scattering center approach is utilized. In this region



collective interactions are very weak so that the body is treated as a collection of independent scattering centers. Detailed geometries now become important in the scattering process. The net scattering from the body is the complex phasor sum of all the individual scattering centers. True optics scattering is defined in the limit as  $A \rightarrow 0$ . For most cases of interest to us, however, we must still deal with finite body size effects. The following list typifies scattering mechanisms in what we have called the optical regime:

- Specular scattering: This is true optics scattering in the sense of  $A \rightarrow 0$ . It is the ray optics case of angle of reflection equal to angle of incidence. The scattering is the optics mirror reflection, and it is mechanism responsible for bright spikelike scattering;
- End-region scattering: This is scattering from the end regions of finite bodies, which produces sidelobe scattering in directions away from specular;
- Diffraction: This is end-region scattering in the specular direction due to edge-induced currents at leading or trailing edges, tips, or body regions of rapid curvature change;
- Multiple-bounce: This is the separate case of mutual body interaction in the sense that one body surface



specularly scatters energy to another body surface that is orientated to reflect this energy back to the observer; for example, corner reflectors and cavities.

### 3.5 ELECTROMAGNETIC THEORY

Radar cross section analyses require a knowledge of the electric field  $E$  or magnetic field  $H$  of the incident wave as it interacts with a scattering body. This section reviews the laws governing electromagnetic phenomena, which are collectively called Maxwell's equations, to develop the wave equation and discuss waves at boundaries. To more fully understand the physics of the EM wave, we must start with Maxwell's equations, which are the fundamental laws governing all electromagnetic behavior.

#### 3.5.1 Source Quantities for Fields and Maxwell's Equations

There are four fundamental field quantities and four fundamental source quantities. The two electric field quantities are

$E$  = electric field intensity, in volts/meter;

$D$  = displacement flux, in coulombs/meter<sup>2</sup>; the two magnetic field quantities are

$H$  = magnetic field intensity, in amperes/meter;

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

$B$  = magnetic induction flux, in Tesla or Webers/meter<sup>2</sup>;

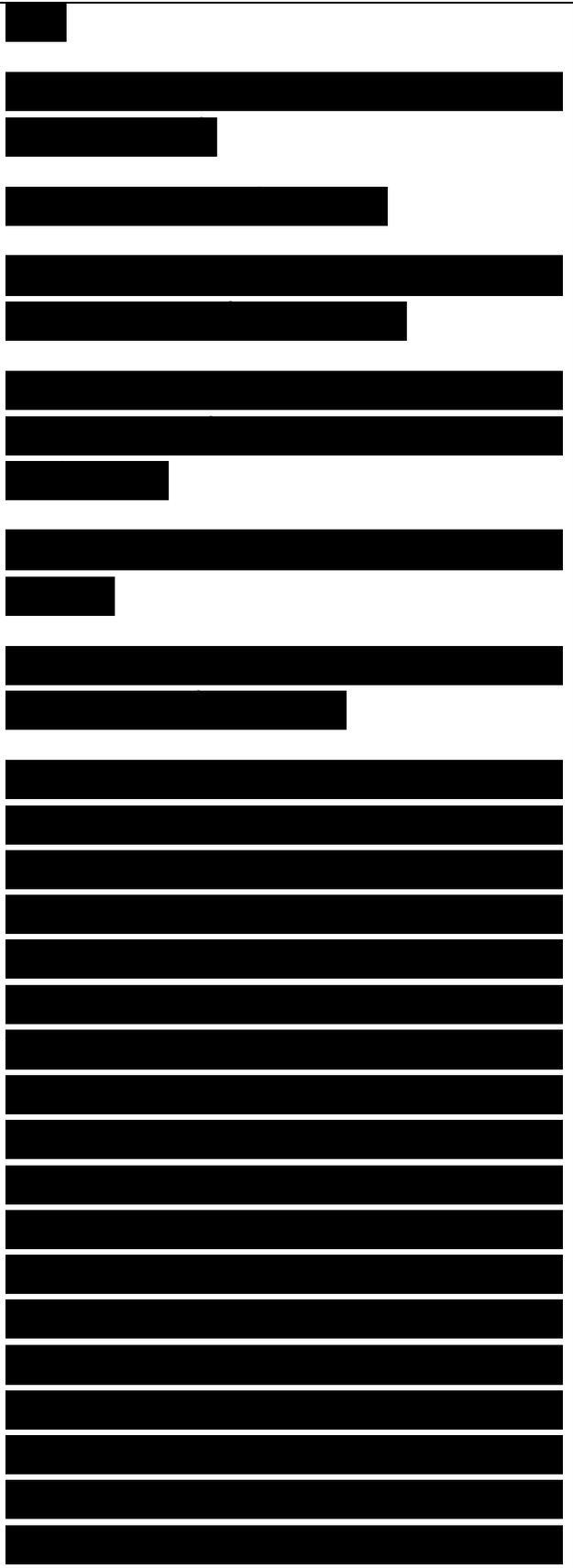
The two electric source quantities are  
 $\rho$  = electric charge density, in coulombs/meter<sup>3</sup> (a scalar);

$J$  = electric current density, in amperes/meter<sup>2</sup> (a vector); two magnetic source quantities are

$\rho^*$  = fictitious magnetic charge density (a scalar);

$M$  = fictitious magnetic current density, in volts/meter<sup>2</sup>, (a vector).

All electromagnetic behavior is governed by a set of four equations known as Maxwell's equations, which relate these field and flux variables among themselves and to sources. These equations totally summarize electromagnetic behavior, and they are usually expressed in differential form. However, they can also be expressed in integral form. Maxwell is associated with these laws because he completed the set by recognizing the need to add a displacement term, which then predicts the propagation of EM waves, as later shown experimentally by Hertz. Three of the four physical relationships are also known under the separate names of Gauss, Faraday, and Ampere.



Maxwell's equations specify the divergence and curl of the vector field quantities. They are sufficient to completely characterize the field. Conservative vector fields start and end on source charges and are described by scalar potentials. Solenoidal fields close back on themselves and are described by vector potentials. EM fields, which are not static, have both conservative and solenoidal components. Near scattering bodies the field lines originate from the surface charges and currents and are mostly conservative. Far EM fields are solenoidal; for example, a propagating spherical wave.

Gauss's law (Fig. 3.18) is a statement relating electrical displacement flux to its source the electrical charge density. This law is a restatement of Coulomb's law:

$$\nabla \cdot \mathbf{D} = \rho \quad \oint \mathbf{D} \cdot d\mathbf{S} = \int \rho dV = q$$

where  $\mathbf{D} = \epsilon\mathbf{E}$ . Physically this tells us that electric field lines originate on electric charge and that the value of electric field summed over a closed surface is proportional to the enclosed charge. This is the "divergence" specification for the field and is the conservative part.

Faraday's law (Fig. 3.19) is the

relationship specifying the solenoidal part of the electric field. This tells us that the solenoidal part is caused by a time rate of change of magnetic field:

(3.34)

where  $\langle P$  is the magnetic flux through the open surface  $S$ . This tells us that the amount of voltage induced around a closed loop is proportional to the time rate of change of enclosed magnetic flux.

Ampere's law (Fig. 3.20) is the relationship saying that a magnetic field may also have solenoidal components and that this component is caused by an electric current density  $J$  and a time-changing displacement current  $dD/dt$ . This last term was Maxwell's contribution, leading immediately to self-propagating electromagnetic waves (time-changing  $E$  causes  $H$  and a time-changing  $H$  causes  $E$ ):

Dipole, Adjacent Positive and Negative Charges

Gauss's Law:  $\nabla \cdot E = \rho/\epsilon_0$   $\oint E \cdot ds = q/\epsilon_0$   $D = \epsilon_0 E$

Relates source charge density to field strength  $E$ . Charge is related to scalar potential and the conservative component of the electric field.

Figure 3.18. Gauss's law: Electric charge as the source of electric fields.



This tells us that the amount of magnetic intensity induced around closed loop is proportional to the current and time rate of change of  $D (= eE)$  through the loop. The static form of Ampere's law is the Biot-Savart law of magnetostatics.

The last of Maxwell's equations (Fig. 3.21) does not have a name. It is the source statement for the conservative part of the magnetic induction  $B$ :

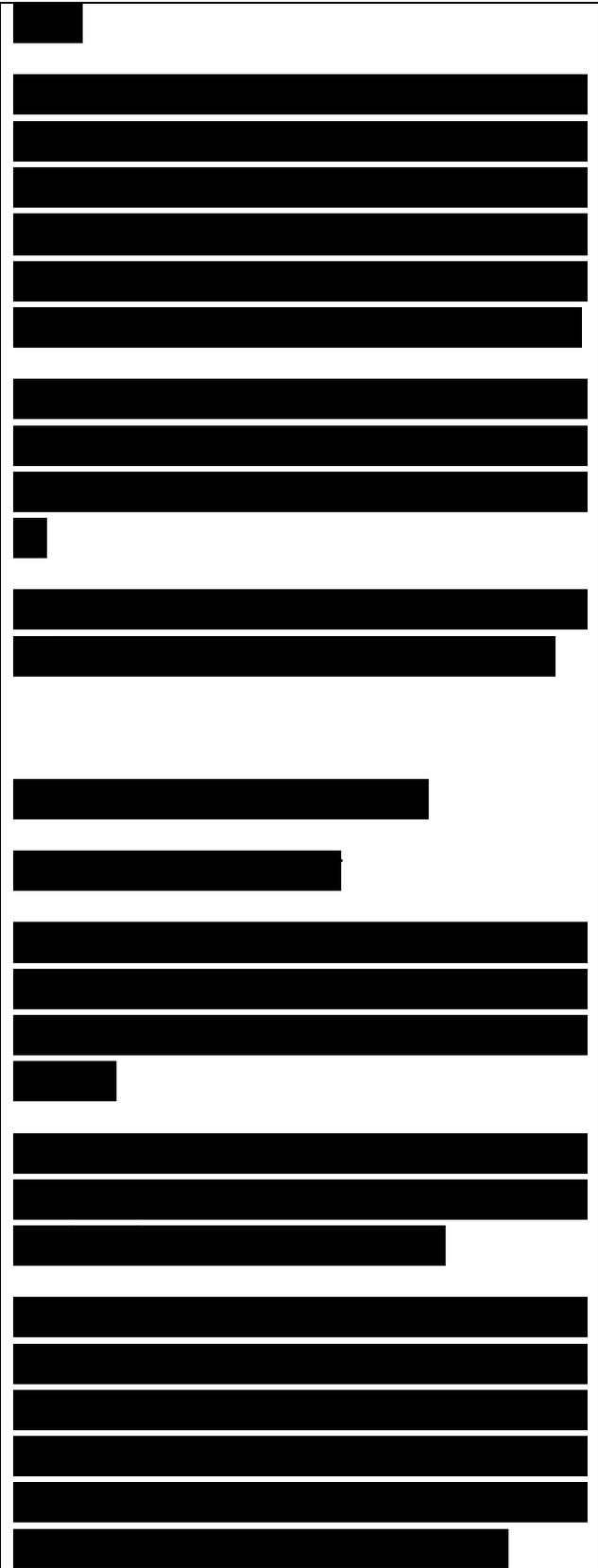
Direction of  $E$  is to generate a current which would oppose the flux  $B$  (Lenz's Law)

Faraday's Law:  $\nabla \times E = -\frac{dB}{dt}$

Time Changing  $B$  is related to the vector potential and the solenoidal (non-conservative) component of the electric field.

Figure 3.19. Faraday's law: Time-changing magnetic field as the source of electric fields.

This relation states that there is no conservative part to the magnetic induction  $B$ ; that is,  $B$  field lines are entirely solenoidal and must always close back on themselves. They do not begin or end on any form of charge density.



These four physical laws, (3.33) through (3.36), summarized in Figures 3.18 through 3.21 and in Table 3.2, along with the boundary conditions on the fields at interfaces form the mathematical-physical basis for all electromagnetic phenomena.

### Material Body Influence

Material bodies influence field quantities because they can store electric and magnetic energy in the microscopic structures of the bodies; that is, induced or permanent dipole moments.

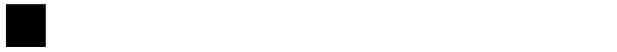
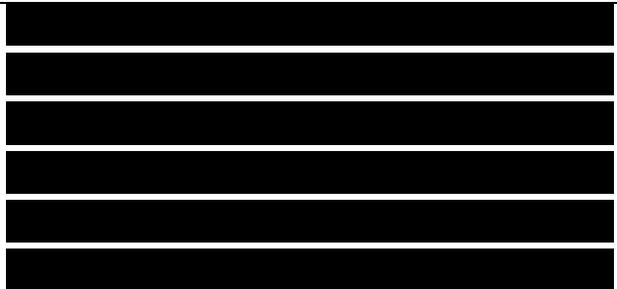
### Magnetic Intensity H due to Enclosed Current J and Time Changing D (- eE)

Ampere's Law:  $\nabla \times \mathbf{H} = \mathbf{J} + \frac{d\mathbf{D}}{dt}$   
= Total Current Enclosed

Current J and time changing displacement D are related to the solenoidal (non-conservative) component of the magnetic intensity H.

Figure 3.20. Ampere's law: Current density J and time-changing displacement D as sources of the magnetic intensity H.

Electric field E and displacement flux D are related to the material electric polarization P by  
 $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$



Sôlênôit

Table 3.2

Summary of Maxwell's Equations (in a vacuum)

I. Sources for displacement  $D$  and electric field  $E$  are

1. Electric charge  $\rho$
2. Time-changing magnetic field  $B$

II. Sources for magnetic flux  $B$  and intensity  $H$  are

1. Electric current  $J$
2. Time-changing  $D$  ( $= \epsilon_0 E$ )

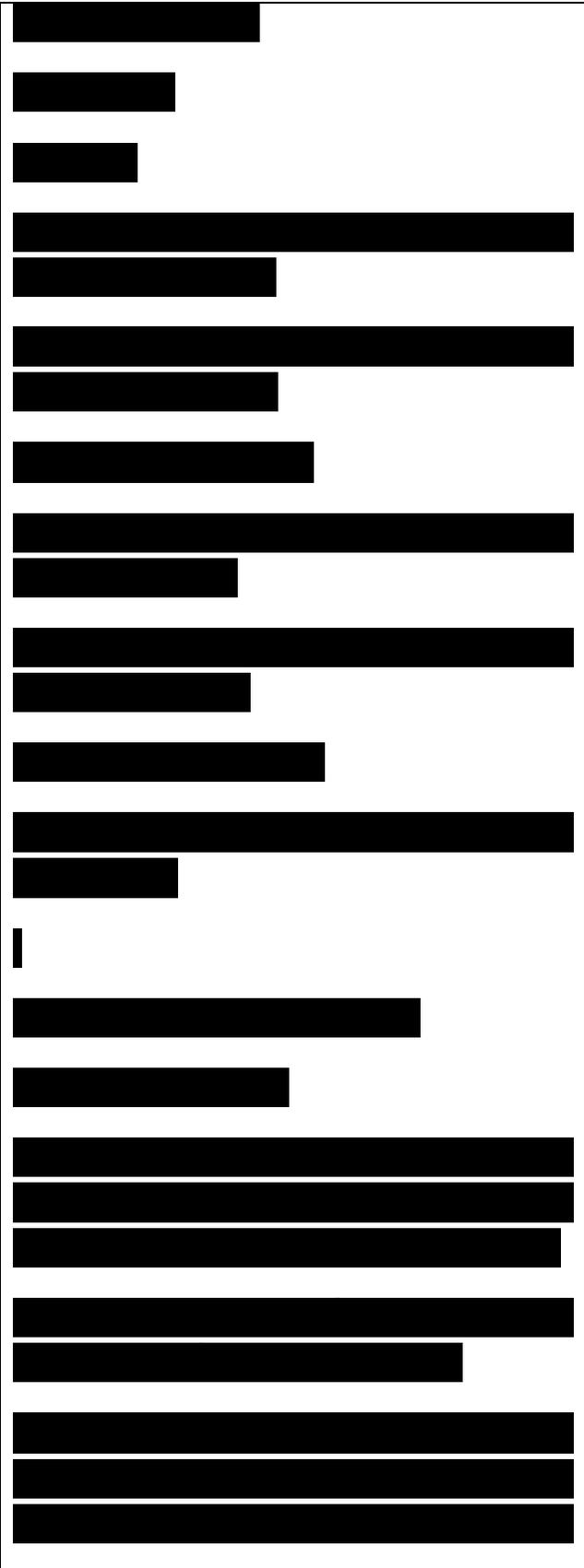
Lines of  $B$  close on themselves

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

Absence of charge as a source for  $B$  means that  $B$  is entirely a solenoidal, non-conservative field.

Figure 3.21. Absence of magnetic charge as source for magnetic field  $B$ .

where  $\epsilon_0$  is the permittivity of free space. Usually material polarization  $P$  is a linear function of the electric field,  $P = \chi E$ , where the proportionality constant  $\chi$



is called the electric susceptibility.

Hence we have

$$D = \epsilon_0 E + P = \epsilon_0 E + \epsilon_0 \chi_e E \quad (3.38)$$

which defines the relative dielectric constant  $\epsilon_r$ , which is a nondimensional parameter (note that  $\epsilon_0$  is in units of farads/meter).

Magnetic field  $H$  and flux induction  $B$  are related by the material magnetization  $M$ :

$$B = \mu_0 (H + M) \quad (3.39)$$

where

$\mu_0$  is the permeability of free space. In general,  $M$  is a nonlinear function of  $H$  or  $B$  as given by a conventional  $B$ - $H$  or  $M$ - $H$  curve for magnetic materials and, as such, exhibits the effect known as hysteresis. Many classes of material, however, may be characterized as isotropic as well as linear so that we have

$$M = \chi_m H \quad (3.40)$$

where  $\chi_m$  is the magnetic susceptibility. Then we have

$$B = (1 + \chi_m) \mu_0 H = \mu_r \mu_0 H$$

where the nondimensional parameter  $\mu_r = 1 + \chi_m$ , and this is called the relative magnetic permeability (note that  $\mu_0$  is in units of henrys/meter). Compared to  $\epsilon_r$ ,  $\mu_r$  may take on rather large values in magnetic materials such as iron and nickel. A word of caution, the concept

of  $\mathbf{j} \times \mathbf{r}$  requires careful justification when working with magnetic materials.

Finally, we have Ohm's law, which relates electric field  $\mathbf{E}$  to current density

$\mathbf{J}$ :

$$\mathbf{J} = \sigma \mathbf{E}$$

where the constant of proportionality  $\sigma$  is the conductivity of the medium and is in units of Siemens/meter. (In older literature,  $\sigma$  was expressed as mho/m, where "mho" is "ohm" spelled backward.

The preceding material relationships are collectively called the constitutive equations:

$$\mathbf{D} = \epsilon \mathbf{E} \quad \mathbf{B} = \mu \mathbf{H} \quad \mathbf{J} = \sigma \mathbf{E}$$

### 3.5.2 Electromagnetic Scalar and Vector Potentials

EM vector fields (indeed, all vector fields) are described by their sources. These can take either or both of two forms. One source is a charge density from which field lines start and end. This is the conservative part of the field, its divergence. The second source relates to how or if field lines close back on themselves. This is the solenoidal part, and it is related to the field's curl (rotation). By specifying both the flux source, with divergence, and the solenoidal source, with curl, we can completely specify the EM field in terms of its charge and current sources.

The use of potential functions in electromagnetics is an alternate way of expressing the fields in terms of their scalar and vector sources in an intermediate fashion. Scalar potential functions express the nature of the scalar charge density as the source of the conservative part of the EM field whereas the vector potential expresses the nature of the vector current source densities as the source for the solenoidal part of the EM field. Maxwell's equations can then be applied to derive relationships that the potential functions must satisfy. The utility of these potential expressions is mostly for theoretical understanding of the sources for vector field.

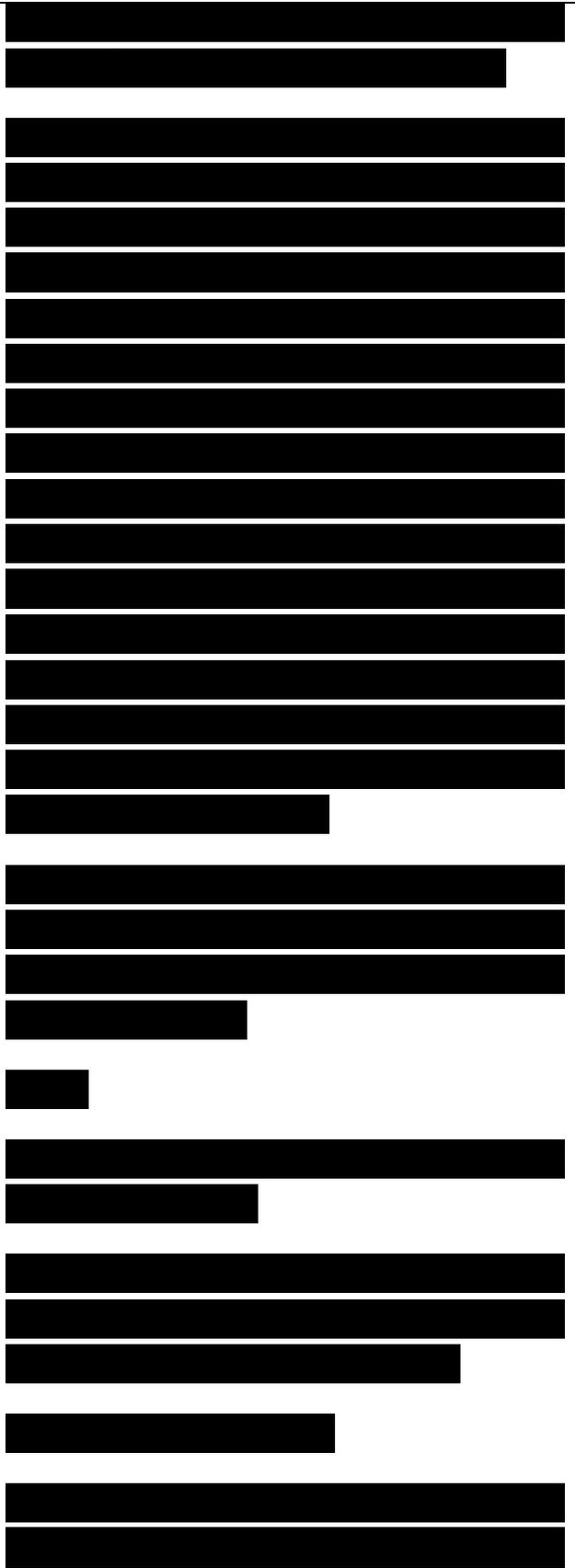
Let us state the standard results. The scalar potential function is an integral sum over an electric charge density with the free-space Green's function:  
(3.44)

This potential is the "source" for the conservative part of the E field.

The vector potential function is an integral over a current density (a vector quantity) with the free-space Green's function:

$$A(\mathbf{r}) = \int \mathbf{J}(\mathbf{r}') \frac{dV'}{|\mathbf{r} - \mathbf{r}'|} \quad (3-45)$$

The magnetic field  $\mathbf{B}$  is completely solenoidal, since  $\nabla \cdot \mathbf{B} = 0$ , therefore it



is expressed entirely in terms of the vector potential A:

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \quad (3.46)$$

which shows that the magnetic field B has only electric current as its source. The electric field E, has both conservative and solenoidal parts, it is expressed by using both potential functions:

$$\mathbf{E}(\mathbf{r}) = -\nabla \phi - \dot{\mathbf{A}} \quad (3.47)$$

which shows that the E field has electric charge and current as its source.

This last expression is of great use in numerical computations for the scattered field due to currents induced on a scattering body. In the far field the E field lines are solenoidal and expressed as due to only the transverse components of A. Therefore if we know J either by a numerical computation or an assumption such as physical optics, then the scattered field is simply found as

$$\mathbf{E}_{\text{scat}}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \nabla \times \nabla \times \int \frac{\mathbf{J}(\mathbf{r}', t - r/c)}{r} dV' \quad (3.48)$$

$\frac{1}{4\pi\epsilon_0} \nabla \times \nabla \times \int \frac{\mathbf{J}(\mathbf{r}', t - r/c)}{r} dV'$

This expression is sometimes called the radiation integral, because it tells us directly the scattered field in terms of the source currents.

Rather than solving for the potential functions and then the fields by the appropriate gradient and curl differential vector operations, modern numerical methods usually solve the integral or

differential Maxwell's equations directly.

### 3.5.3 Wave Equation

Maxwell's equations, after the addition of the term by Maxwell and for whom the entire set is named, predicts or allows EM fields to propagate on their own away from charge and current sources. It is instructive to derive the wave equation and examine the nature of this wave that it predicts. Before we jump into the usual derivation, let us first reflect on the fact that Faraday's law tells us that a time changing  $B( = \nabla \times H)$  field is the solenoidal source for  $E$  and that Ampere's law, as amended by Maxwell, tells us that a time-changing  $D( = \nabla \times H)$  is the solenoidal source for  $H$ . Therefore, a time-changing  $H$  causes  $E$ , and a time-changing  $E$  causes  $H$ .

An electromagnetic wave "is its own source," and hence becomes self-propagating.

We start the derivation by assuming time harmonic fields expressed in complex phasor notation where the actual field is the real part of the complex quantity:

$$E(r,t) = \text{Re} \{ E(r) e^{j\omega t} \} \quad (3.49)$$

The choice of  $(+ j\omega t)$  follows the electrical engineering convention, as the

alternate choice of  $(-\omega t)$  would follow the physics convention. This choice, although arbitrary, has an impact on the choice for the free-space Green's function:

In either choice, the physical fields are the real part:

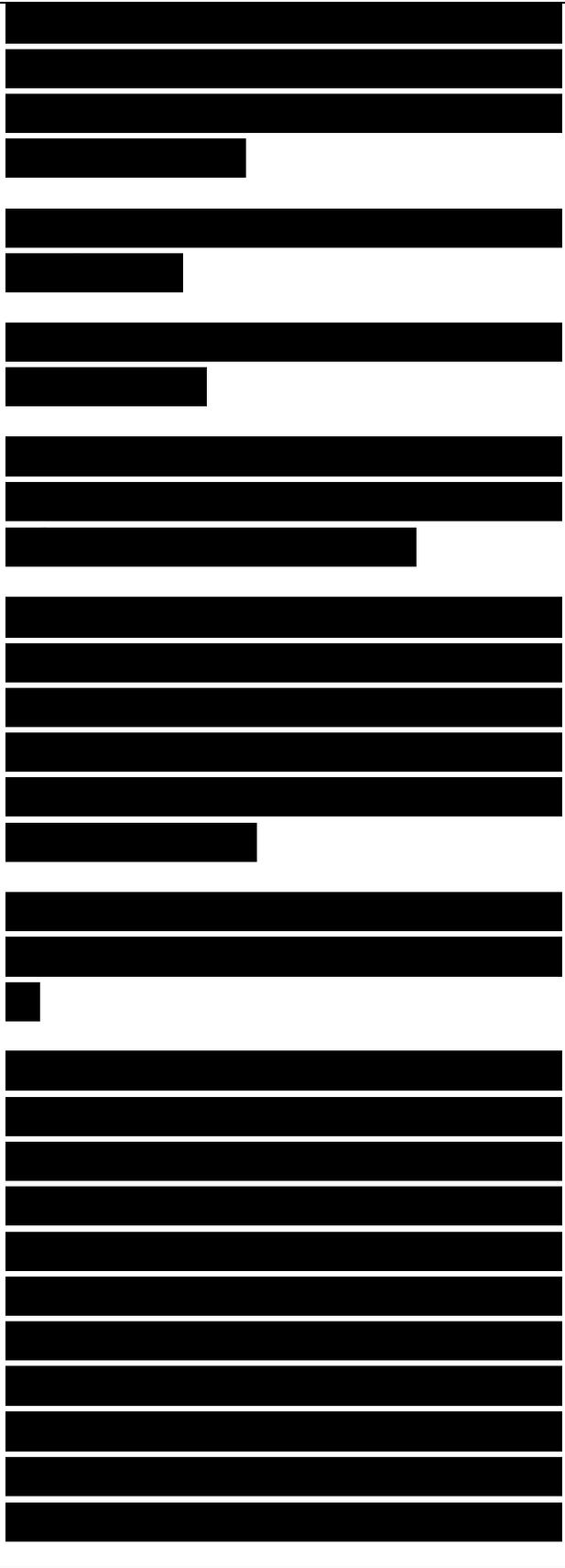
$$E_{\text{physical}}(\mathbf{r}, t) = \Re\{E e^{j(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi)}\} = E_a \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi) \quad (3.51)$$

where  $\phi$  is the phase of the indicated vector component of  $E$  (each component has its own phase).

Maxwell's equations then take the following form for this time-harmonic assumption and the constitutive relations, using  $+j\omega$ . The wave equation is obtained by taking the curl of Faraday's law to obtain

The curl is expanded for charge-free region (away from charge sources) where

We note that to obtain the wave equation we have taken the curl of the curl of a vector field. Because the curl is a measure of rotation, we compute the "rotation of the rotation," a type of second derivative or acceleration term. We shall see that this "acceleration" of the field (when  $\mathbf{a} = \mathbf{0}$ ) is proportional to the negative of itself, and hence, an oscillatory solution is obtained. This occurs in mechanics when the force is opposite to the direction of motion. When  $\mathbf{c} \neq \mathbf{0}$ , the acceleration has



another term,  $-j\omega\epsilon_0 E$ , which leads to damping of the wave due to resistive losses.

The wave equation for the electric field is finally obtained by using (3.53) in (3.54):

$$\nabla^2 E = j\omega\epsilon_0 E - \alpha \text{Pfx}E \quad (3.55)$$

for free space or dielectric media, where  $\alpha = 0$ , and for  $k^2 = \omega^2 \epsilon_0 \mu_0 = (2\pi/\lambda)^2$ , we obtain

$$\nabla^2 E + k^2 E = 0 \quad (3.56)$$

The solution of this wave equation can take several forms depending on the coordinate system. The first is a plane wave in rectangular coordinates (nonphysical because spherical spreading is not allowed):

$$E(r,0) = E_0 e^{-i(k \cdot r - \omega t)} \quad (3.57)$$

which corresponds to a wave propagating in the  $k$  direction with wavelength  $\lambda$  and radian frequency  $\omega$ . Its associated magnetic field is obtained from (3.34) in Faraday's law to obtain

which shows us that  $E$ ,  $H$ , and  $k$  are mutually orthogonal. The ratio of  $E$  to  $H$ , the wave impedance, is found by identifying

where  $Z$  is the intrinsic impedance of the medium. With this definition the wave equation shows us that

$$ZH = k \times E \quad (3.60)$$

The velocity of propagation is  $v$ , where  $c$  is the velocity in the absence of material media,

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

A summary of the wave nature of electromagnetic fields is shown in Table 3.3.

The constant  $E^\circ$  term in the wave equation solution must be orthogonal to  $k$ . This means that  $E$  is arbitrary to the extent that it must reside in a plane perpendicular to  $k$ . The actual direction of  $E$  is called polarization and is determined by the sources that launched the wave (e.g., the antenna).

Plane wave concepts are convenient for analytical purposes, but waves in the real world are spherical. Their magnitudes fall off inversely with distance from their localized source point:

so that the energy flux,  $E^2$ , decays as  $1/r^2$ . For a two-dimensional coordinate system with infinite line sources we can also have a nonphysical cylindrical wave of the form

which has an energy flux decay of  $1/r$ . The plane wave, although not physical, is a very useful concept because a

spherical wave at very large distances does indeed appear to a local target as a plane wave because target dimensions would be much smaller than the wave curvature. Therefore a spherical wave from a distant radar incident on a scatterer can be treated as a plane wave.

Table 3.3

Wave Equation Summary (in a vacuum)

Maxwell says;

$\nabla \times \mathbf{H} = \mathbf{j} + \dot{\mathbf{D}}$  is related to time-changing  $\mathbf{D}$   
( $= \epsilon_0 \mathbf{E}$ )

$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$  is related to time-changing  $\mathbf{B}$   
( $= \mu_0 \mathbf{H}$ )

The wave equation is derived from Maxwell's equations and shows that an electromagnetic wave has the following characteristics:

$\mathbf{E}$  and  $\mathbf{H}$  are wavelike, oscillating in time and space.

An EM wave has both  $\mathbf{E}$  and  $\mathbf{H}$  components.

$\mathbf{E}$  is perpendicular to  $\mathbf{H}$ .

$\mathbf{E}$  and  $\mathbf{H}$  are perpendicular to the direction of propagation  $\mathbf{k}$ .

Velocity of propagation  $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$ , the speed of light in free space.

$\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$

(Z.

[REDACTED]

Ratio of E to H is the wave impedance  
 $Z_0 = 377\Omega$  for free space.

In a plane perpendicular to  $k$ , E is arbitrary, hence the concept of polarization.

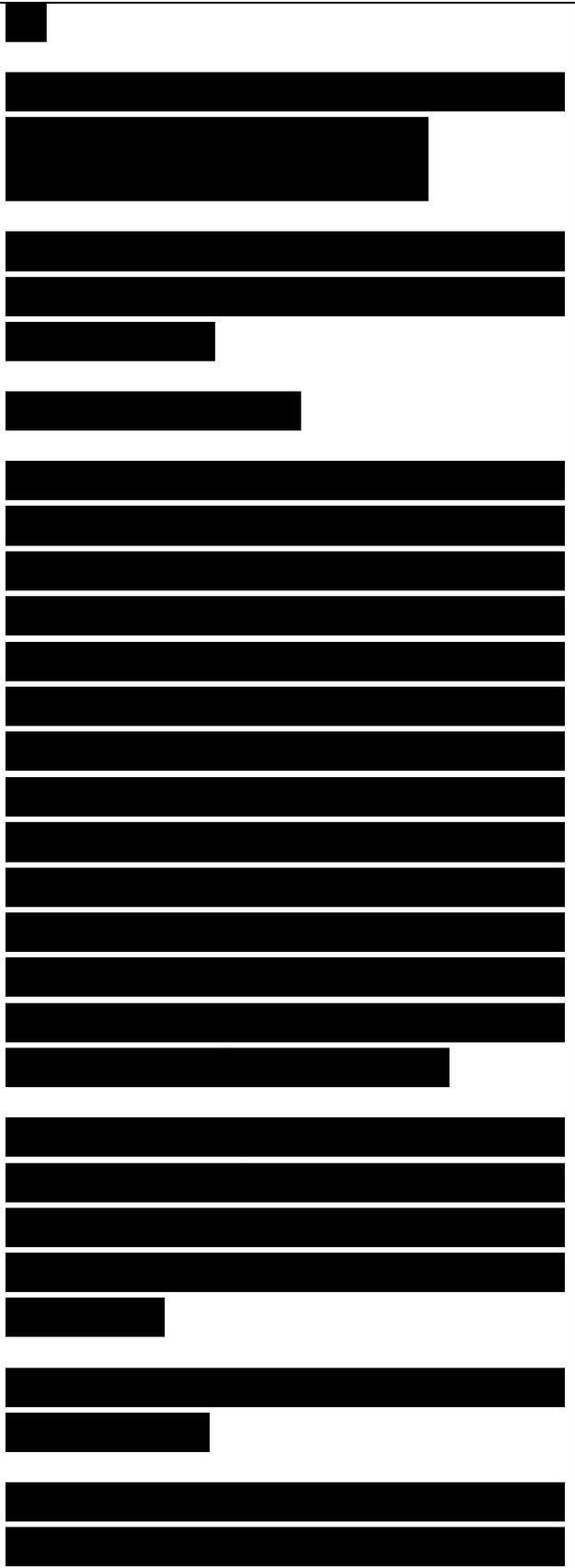
### 3.5.4 Waves at Boundaries

Radar cross section phenomena is inherently a study of what happens when an EM wave strikes a boundary. This study involves boundary conditions and reflection coefficients. Based on the physical reasoning that a charge can be neither created nor destroyed and that electromagnetic fields are created by charge and current distributions, the fields must satisfy certain boundary conditions at the interface between two different media. The conditions can be derived from the integral form of Maxwell's equations applied to an interface.

The derivation for the surface perpendicular B and D fields require the use of a small "pillbox" volume and the respective divergence equations in integral form.

The normal components of B must be continuous; therefore,

where  $n$  is a unit normal outward from the surface. The normal component of D



is discontinuous by the amount of free surface charge density,

Tangential field boundary conditions on E and H are derived using the two curl equations in integral form (Faraday's and Ampere's laws) using a small rectangular loop at the interface. This leads to the requirement that the tangential E field be continuous across the boundary:

and that the tangential components of the H field be discontinuous by the amount of surface current density J:

The surface current can have a nonzero value when the integration loop is reduced to zero only when  $J$  is infinite. This requires that the conductivity  $\sigma$  be infinite,

meaning that the surface must be perfectly conducting. For finite conductivity, the tangential magnetic field is continuous across the boundary:

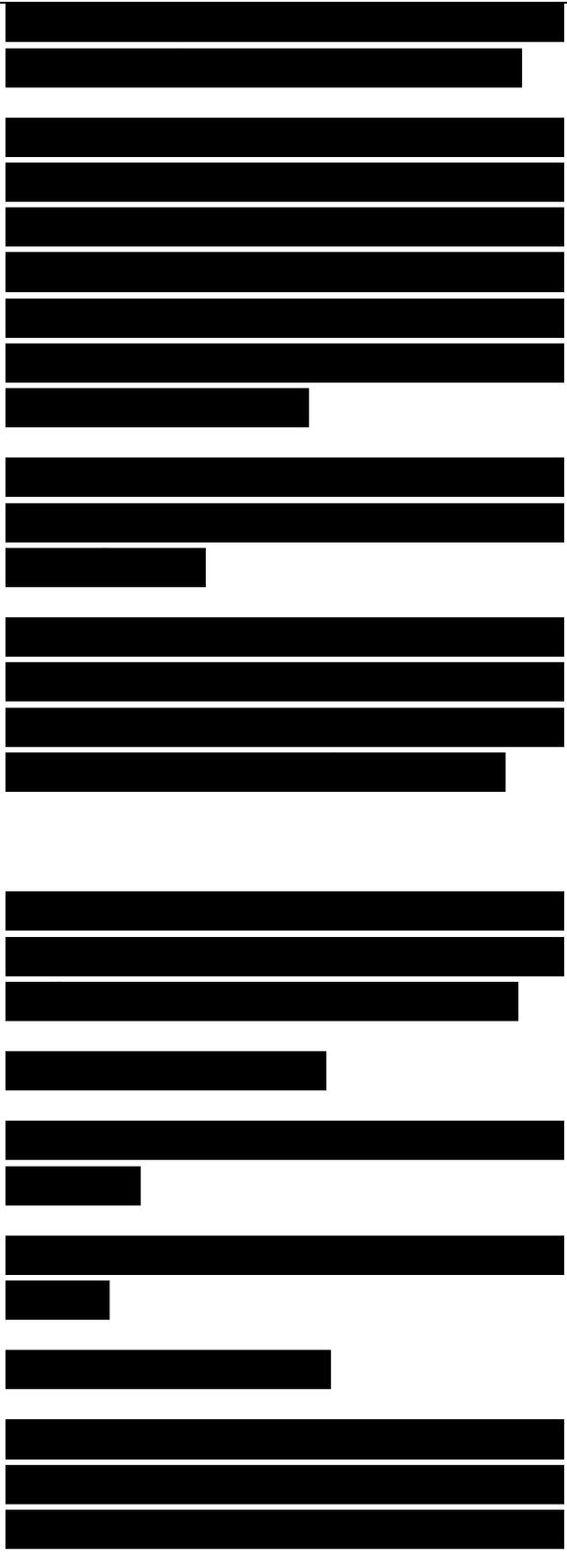
$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s \quad (3.68)$$

A summary of boundary conditions is given in Figure 3.22.

Figure 3.22. Summary of electromagnetic boundary conditions.

### 3.5.5 Reflection Coefficients

No study of RCS can be complete without a discussion of reflection coefficients. These help to describe specular scattering from surfaces, which



is the major scattering mechanism for most targets. However, they do not necessarily apply to other scattering mechanisms, such as end-region, **edge diffraction**, and surface wave scattering.

When a wave impinges on an interface, part of the wave is reflected and part is transmitted, Figure 3.23. At the interface, the incident, reflected, and transmitted wave have the same phase:

$$\phi_{\text{incident}} = \phi_{\text{reflected}} = \phi_{\text{transmitted}}$$

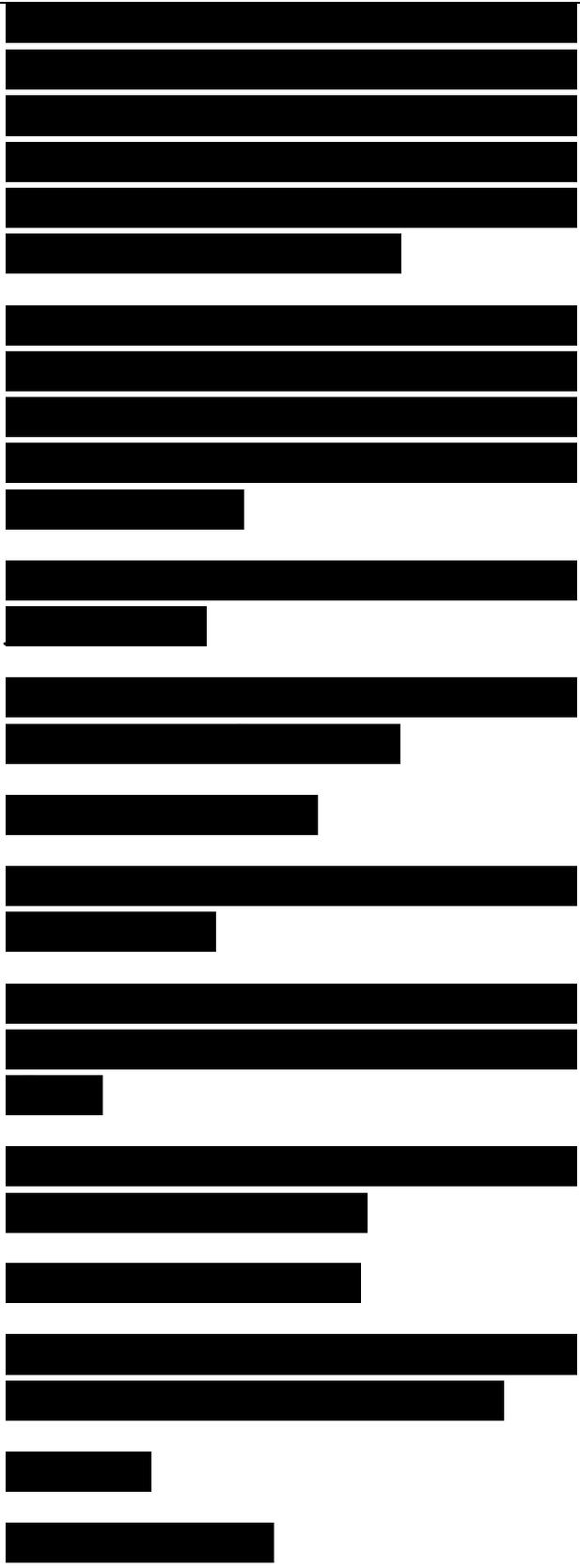
This leads to Snell's law, which requires the angle of reflection be equal to the angle of incidence,  $\theta_{\text{reflected}} = \theta_{\text{incident}}$

Figure 3.23. Reflection and transmission at an interface.

This specular scattering relationship is the central concept for high-frequency optics regime scattering.

The angle for the transmitted ray may be computed from (3.69) as  $k \sin \theta_t = k_2 \sin \theta_i$

which may be recast in terms of the index of refraction  $n$  of each medium (as  $k = n\omega/c$ ):  $n_1 \sin \theta_t = n_2 \sin \theta_i$



where  $n_1$  and  $n_2$  are the indices of refraction of the two media separated by the interface. When a wave passes from air,  $n_1 = 1$ , to a more dense medium,  $n_2 > 1$ , the wave slows and bends toward the normal:

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

The bending of rays in passing from one medium to another is known as refraction. Because the bending is governed only by the electromagnetic properties of the media on either side of the interface, the refractive indices are sufficient to characterize the mechanism.

### Reflection and Transmission Coefficients

The amount of specular reflection and transmission of electric and magnetic fields at a plane interface can be expressed in terms of the Fresnel reflection and transmission coefficients associated with the interface,  $R$  and  $T$ . These are functions of polarization, incident angle, and material properties. In general if  $E_i$ ,  $E_r$ , and  $E_t$  are the incident, reflected, and transmitted fields at the interface, the reflection, and transmission coefficients are defined as field ratios (not power),

$$(3.74)$$

These are functions of polarization, electromagnetic material parameters ( $\epsilon$ ,  $\mu$ ) for each medium, and the angle of incidence. When the  $E$  field is polarized

perpendicular to the plane of incidence, the boundary conditions require that [6],

and when the incident E field is polarized in the plane of incidence we have  
(3.76)

where the values of Z and k are the intrinsic impedances and the wavenumbers of the two media. Z may be complex as in general n and e are complex numbers (the complex part is the loss mechanism of the material):  
(3.77)

The wavenumber ratio for nonmagnetic materials reduces to the ratio of the indices of refraction:  
(3.78)

An example of R as a function of incident angle for a  $\epsilon = 1$  to  $\epsilon = 4$  interface, such as air to fiberglass, is shown in Figure 3.24. Note that for parallel polarization R becomes almost zero near  $65^\circ$ . This is called the Brewster angle, where little energy is reflected and most is transmitted into the second medium.

When the incidence angle is normal to the interface,  $\theta_i = 0^\circ$ , and the two reflection coefficients reduce to the same value;



If medium 2 is a very good conductor (metal or seawater at 10 GHz) so that its impedance is very low,  $Z_2 \approx 0$ , then  $R \approx -1$ , meaning that the wave is entirely reflected and suffers a phase change of  $180^\circ$ . If medium 2 has a very high impedance, then  $R \approx +1$ , so that all of the energy is again reflected, but this time without a phase change. If  $Z_1 = Z_2$ , no reflection occurs and the impedance on each side of the interface is the same, which would be the special case for  $\epsilon_r = \epsilon_2$ .

The formalism for EM waves (waves in space) and transmission lines (waves attached to a structure) have a great deal in common. Figure 3.25 lists the analogous relationships.

The reflection and transmission coefficients have been defined for field quantities when no absorption takes place, hence conservation of energy requires that

Angle from Normal (degrees)

Figure 3.24. Fresnel reflection coefficients for an air/ $\epsilon_r = 4$  interface for E parallel and perpendicular to plane of incidence.

When R is expressed using a decibel scale, the relationship is  $R_{dB} = 20 \log_{10}(|R|)$  (3.81)

[Redacted text block]

where we convert field (voltage) to power. The reference value is often taken as unity; that is, a metal surface.

The relationship between R and T is [6]  
 $1 + R = T$  (3.82)

because the tangential components of E must be continuous across the interface and boundary conditions represents **but one point of view**. The second view rec-coefficient becomes

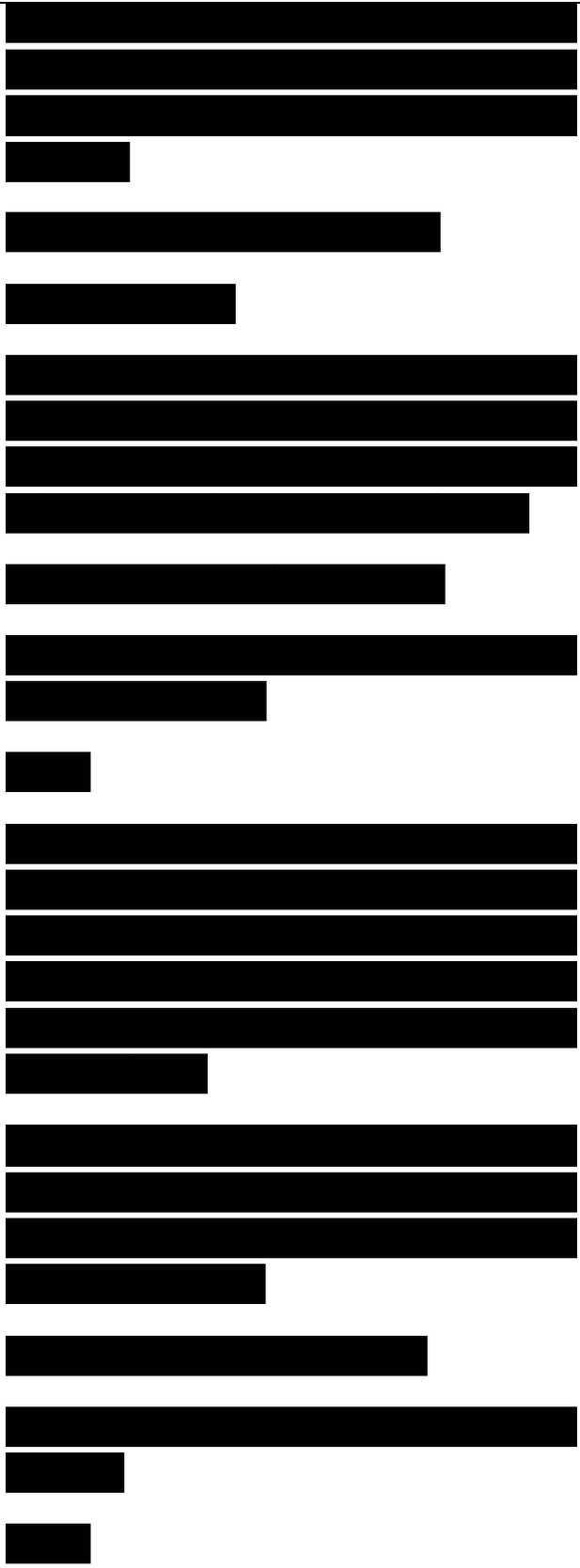
#### Transmission Line EM Wave

Figure 3.25. Transmission line/EM wave analogy.  
(3.83)

The reader is directed to the literature for the case when absorption also needs to be included; for example, when the second medium has nonzero conductivity or a loss tangent.

The dielectric constant  $\epsilon$  can take on complex values if a material has nonzero conductivity. In this case the wave equation has the form  
 $\nabla^2 E + (-\mu\epsilon\omega^2 + a^2/j\omega\epsilon)E = 0$

therefore we identify the wave number  $k$  as a complex number:  
(3.85)



where we identify the permittivity also as a complex number,  
(3.86)

The imaginary part of  $k$  leads directly to absorption of the wave  
(3.87)

For a good conductor,  $\epsilon'' \gg \epsilon'$ , and the complex wave number becomes  
 $k \approx -j\sqrt{\omega\mu\sigma}$

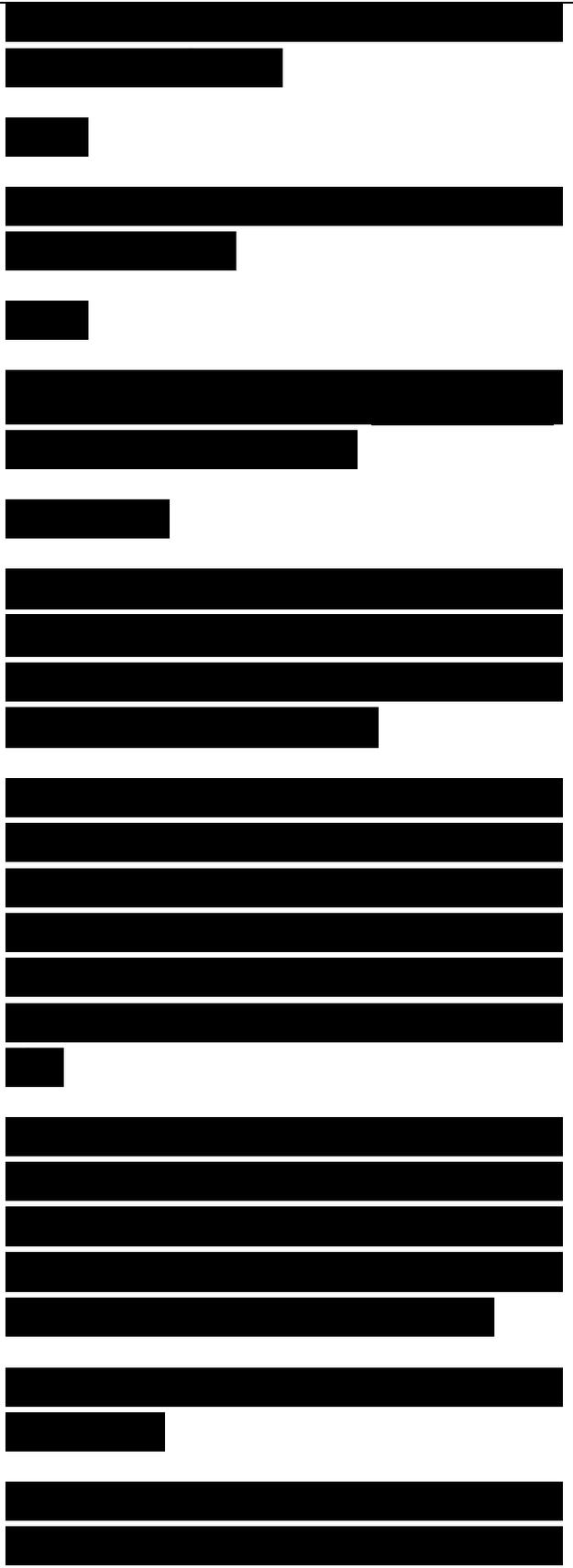
where  $\delta$  is the skin depth and is equal to  $(2/\omega\mu\sigma)^{1/2}$ . Therefore a wave in a good conductor is attenuated by 63% ( $1/e$ ) in a distance  $\delta$ .

These wave reflection formulas show that, if small reflections are desired, such as for RCSR, then the wave should never ever see large changes in impedance. Rather, gradual impedance changes are desired.

In summary, we see that the reflection of wave from plane boundaries depends on polarization, angle of incidence, and EM material parameters of  $(\mu, \epsilon)$ , which, in general, are complex numbers and frequency dependent.

### 3.5.6 Wave Reflection from Surface Current Point of View

The development of reflection coefficients in terms of intrinsic material



impedance and boundary conditions represents but one point of view. The second view recognizes that the scattering processes of reflection and transmission occur because the incident wave induces currents at a material interface. These currents, in turn, produce a scattered wave that reradiates energy in various directions.

In this context, the total field  $H_r$  is the sum of an incident part and a scattered part:

$$H_{\text{Total}} = H_{\text{incident}} + H_{\text{scattered}}$$

The boundary conditions are on the total field. These boundary conditions may be represented as equivalent source currents and charges, which then become the source of the scattered field. The surface electric and magnetic currents are then interpreted in terms of the total tangential fields at the surface. The tangential magnetic field is expressed as an electric current:

$$\mathbf{n} \times \mathbf{H}_r = \mathbf{J}_s \quad (3.90)$$

while the tangential electric field is expressed as a fictitious magnetic current,

$$\mathbf{n} \times \mathbf{E}_T = -\mathbf{M}. \quad (3.91)$$

The corresponding electric and magnetic charge densities are related to their respective currents through the notion of

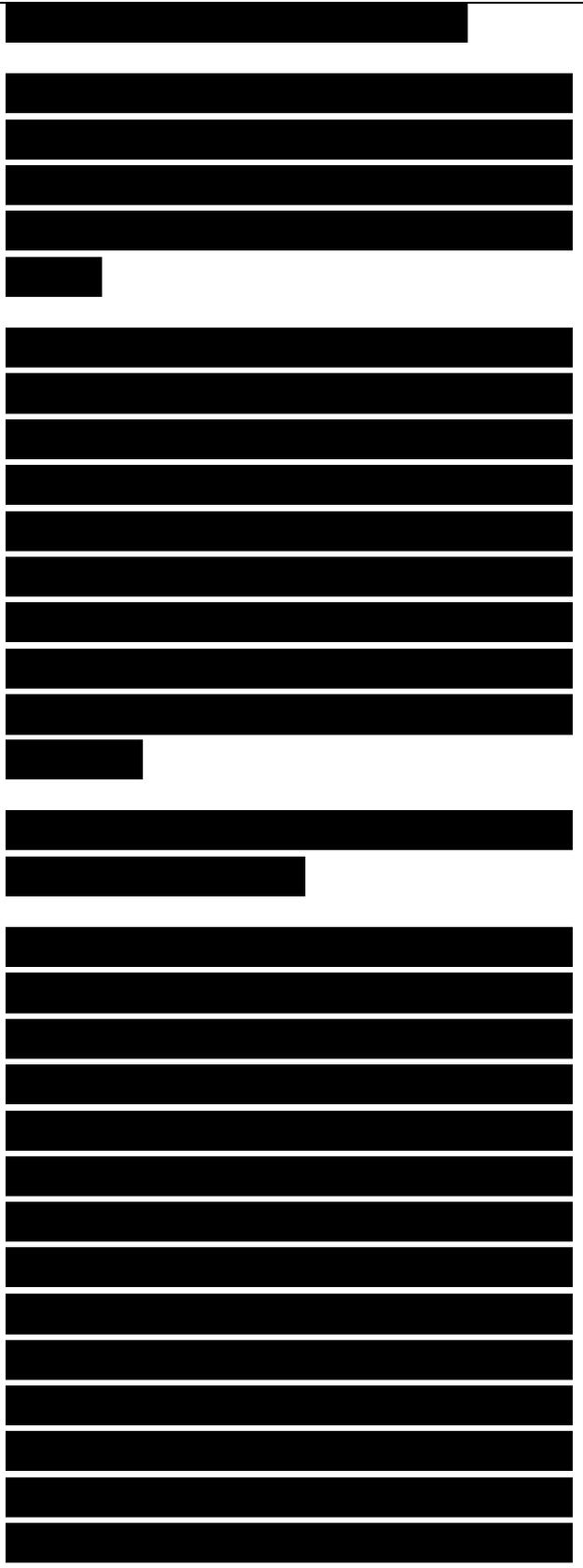
conservation of charge:

For the special case of a perfect conductor, at whose surface the total tangential electric field must be zero, the magnetic current and charge are zero:

In the general case of scattering from electric or magnetic bodies,  $E_r$  and  $H_r$  are finite on an interface; hence, both  $J$  and  $M$  must be considered. As pointed out by Stratton, magnetic currents and charges have never been observed in nature, yet they form a useful mathematical artifice when we enforce arbitrary boundary conditions in terms of induced source currents.

### 3.5.7 Stratton-Chu Equations for the Scattered Field

When several regions of space are involved, we can use Maxwell's equations in conjunction with the vector Green's theorem to arrive at a set of field equations for the scattered field. This has been done by Stratton and Chu, [7]. Consider the geometry of Figure 3.26 where Region I is separated from Region II by a surface  $S$ . Assume that there are magnetic and electric source currents and charges in each region. The field anywhere in Region I is given by the Stratton-Chu equations as the sum of a volume integral over the source in Region I and a surface integral over the fields on surface  $S$  caused by the sources in Region II.



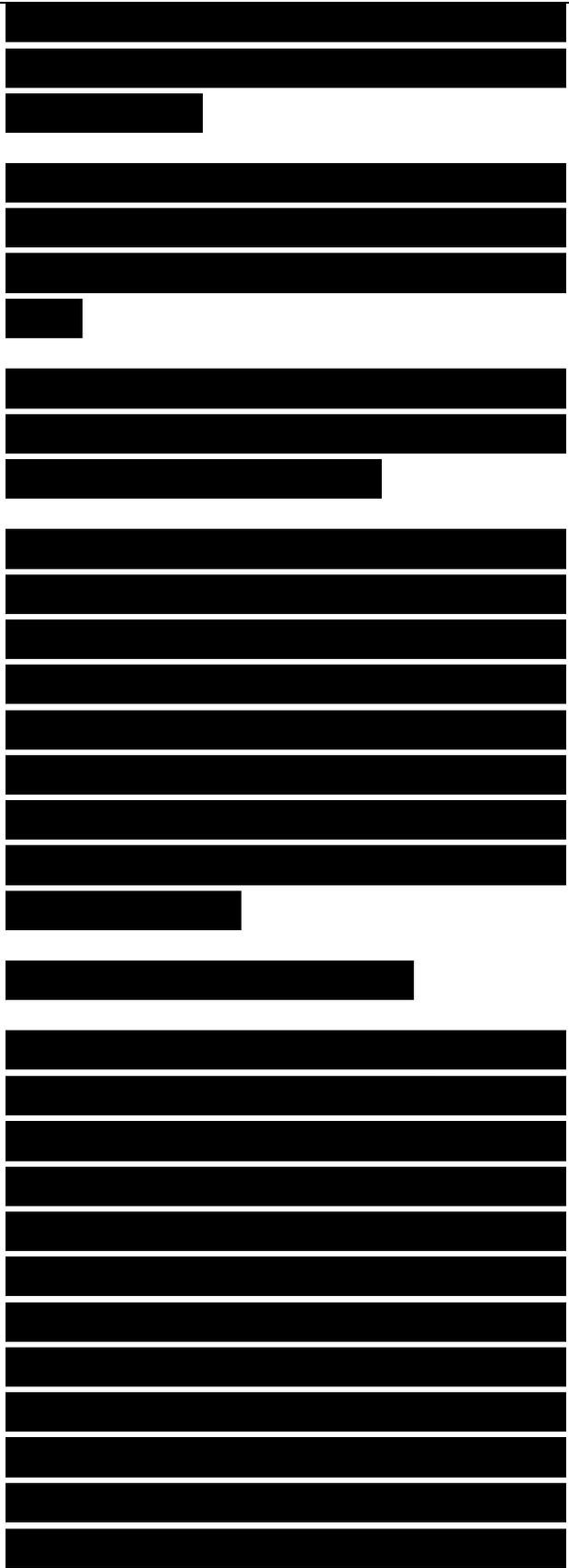
The Stratton-Chu equation for the scattered E field, sometimes called the electric field integral equation (EFIE), is:

and the corresponding expression for the scattered H field, called the magnetic field integral equation (MFIE), is:

The volume integral over the enclosed Region I charge and current sources gives the scattered field due to those sources. The surface integral that separates Region I and II gives the fields in Region I due the sources in Region II. We do not have to know the sources in Region II, only the fields that these sources produce on surface S.

The free-space Green's function

creates the phase delay and spatial  $1/R$  decay between source and field points, where  $R = |\mathbf{r}' - \mathbf{r}_s|$ . These equations are an integral form of Maxwell's equations and are "exact" for any frequency. Their solution requires integral equation techniques. Prior to the computer era, these equations were mostly a curiosity due to the difficulty of analytical solutions. However, the numerical approach known as the method of moments uses these equations as the starting approach for the matrix solutions.



The interpretation that the fields at the surface are the sources in the form of currents and charges is apparent from

The interpretation of the tangential fields in terms of surface currents and perpendicular fields as surface charge is a useful formalism for representing the sources of the fields.

The Stratton-Chu equations describe the general case of scattering from an arbitrary body. For the case of a perfect conductor, the total tangential electric field must be zero,  $n \times E_T = 0$ , so that the magnetic current  $M$  and charge density  $p^*$  are zero. Then, by using the equation for continuity of electric charge (3.94), ( $\omega = ck$ ,  $(\text{Ofi} = kr)$ ], we arrive at the integral equations for the scattered E field:

(3.99)

These equations for a perfect conductor have the following features:

1. Because the current density  $J = \text{fl} \times H_{\text{Total}} = h \times (H_{\text{mc}} + H_{\text{scat}})$ , the scattered field appears under the integral making these integral equations.

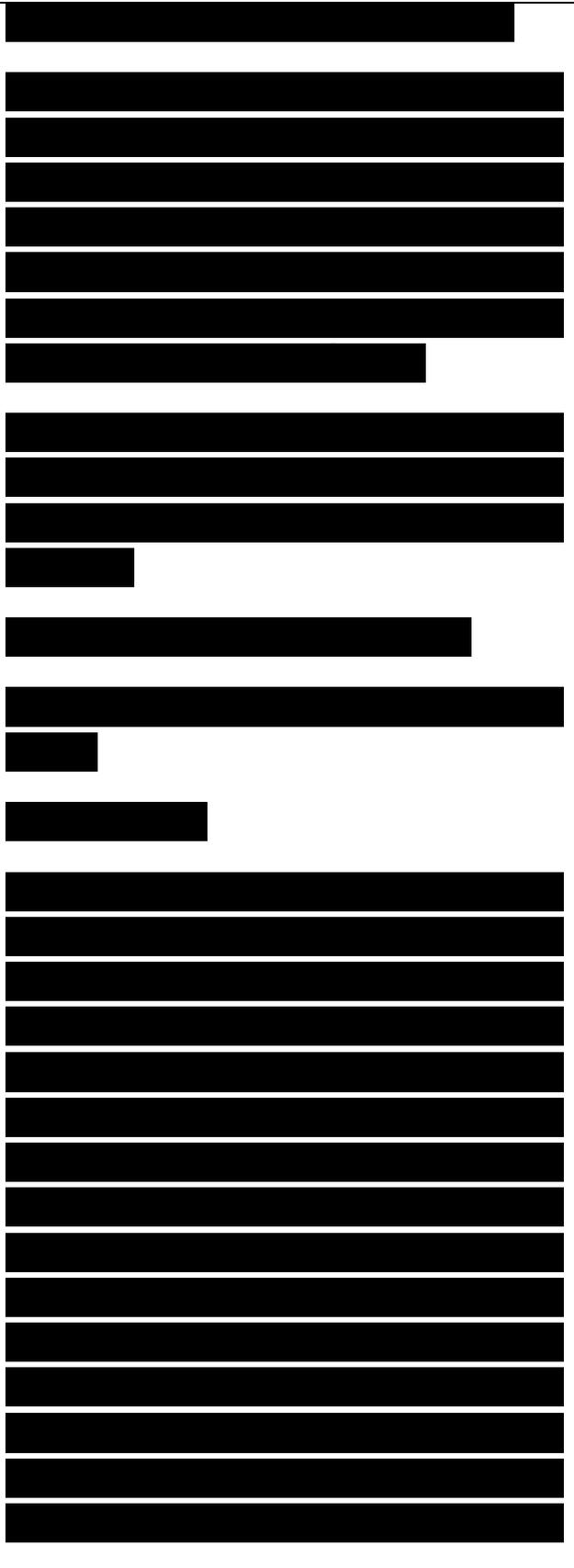
2. In the far field  $E$  and  $H$  are related by the impedance of free space. Therefore we solve either the EFIE or the MFIE, but not both. The only time both need solution is for those rare cases where internal body resonances occur.

3. In the far field, the fields are transverse to the direction of propagation and decay inversely with increasing distance, thus  $a_{112}$  is proportional to  
$$\text{Barfield } a_{jkr} f(A \cdot J)_{ip} dS$$

where  $n^{\circ}$  is the transverse unit polarization vector.

### 3.6 SUMMARY

This chapter has presented an overview of electromagnetic scattering. We have seen that RCS is a measure of power scattered from the incident wave; that it is a function of the angular orientation and shape of the scattering body, frequency, and polarization of the transmitter and receiver. The scattered wave, of which RCS is a measure, is caused by reradiation of currents induced on the scattering body by the incident wave. The scattering process breaks into three natural regimes: the low-frequency or Rayleigh region, where the wavelength is much longer than the scattering body size and the scattering process is due to induced dipole moments where only gross size



and shape of the body are of importance; the resonant region, where the wavelength is on the same order as the body size and the scattering process is due to surface waves (traveling, creeping, and edge) and optics; and the high-frequency optics region, where the wavelength is much smaller than the body and the scattering process is principally a summation of the returns from isolated, noninteracting scattering centers.

Maxwell's equations tell us that EM waves are a combination of electric and magnetic fields that are perpendicular to each other and to the direction of propagation. When an EM wave is incident on a body, the boundary conditions on the fields require that surface currents flow. These currents, in turn, reradiate a scattered EM wave. The strengths of the reflected and transmitted waves for specular scattering are given by the Fresnel coefficients, which are functions of the incident polarization and material properties. Surface fields were shown to be characterized as surface electric and magnetic currents and charges. The formal expressions that



Phương trình Maxwell cho chúng ta biết rằng sóng EM là sự kết hợp của các trường điện và từ vuông góc với nhau và vuông góc với hướng truyền. Khi một sóng EM đến vật thể, các điều kiện biên của trường dẫn đến sự xuất hiện các dòng điện bề mặt. Sau đó những dòng này sẽ tái bức xạ một sóng EM tán xạ. Cường độ của các sóng phản xạ và truyền qua được biểu diễn qua các hệ số Fresnel, những hệ số này phụ thuộc vào độ phân cực tới và tính chất vật liệu. Người ta thấy rằng các trường bề mặt được đặc trưng bởi các điện tích và dòng điện từ bề mặt. Biểu thức tiêu chuẩn có dạng

Đã đánh công thức trong text