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On the Number of Partitions of Sets and Natural Numbers

Abstract

In this article, we first state some relations about the number of partitions of a set under some particular conditions and then we give a new relation about the number of partitions of an n-set, i.e., Bell number B(n). Finally, we give some formulas to count partitions of a natural number n, i.e., partition function P(n).

1 Introduction

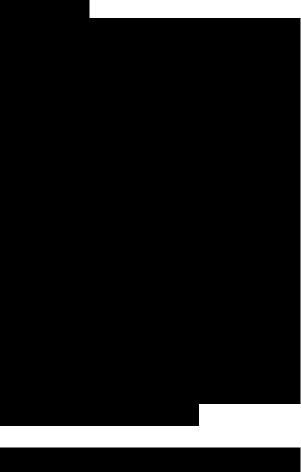
Partitions of sets and natural numbers have been a very attractive subject during the recent decades. Partitions play important roles in such diverse areas of mathematics as combinatorics, Lie theory, representation theory, mathematical physics, and the theory of special functions. Because of some applications of this subject, mathematicians have given some formulas in this regard. Up to now, the number of partitioning of sets and natural numbers was considered by some authors; for more details about partitions of sets and other related subjects, see for example [8], [10], [15], [17], [18], [19], [20], [21], and [22], and for partitions of natural numbers and other related for concepts, see example.....

In this article, using some elementary tools of combinatorial analysis- see [6], [7], [11], [16], and [23]- we give some alternative formulas for theses problems, considering also some special cases.

Về số phân hoạch của các tập hợp và các số tự nhiên

Tóm tắt

Trong bài viết này, trước hết chúng ta phát biểu một số hệ thức về số phân hoạch của một tập hợp trong một số điều kiện cụ thể và sau đó chúng ta đưa ra một hệ thức mới về số phân hoạch của tập n, tức là, số Bell, B (n). Cuối cùng, chúng ta đưa ra một số công thức để đếm số phân hoạch của một số tự nhiên n, tức là, hàm phân hoạch P (n).





2 Partitions of sets

Definition 2.1. A partition of a set A is any sequence of subsets $A\setminus$,...,Am of A, such that (J "L1 Ai = A and Ai fi Aj = 0. Vi = j.

The number of partitions of a set of size n (n-set) is called the Bell number, in honor of famous mathematician "Eric Temple Bell" (1883-1960), and denoted by B(n). By convention we agree that B(0)=1. Using Definition 2.1, for n=1,2,3, we have B(1) = 1, B(2) = 2, B(3)=5 and so on.

In the following lemma, we state a relation for the number of partitions of an n-set such that in every partition we have at least a subset with n - j elements, j = 1, 2,..., [], n > 2.

Lemma 2.1. Let $B(n \setminus n - j)$ be the number of partitions of a set with n elements in which there exists at least a subset with n - j elements, j = 1,2,..., []], Then for n > 2, we have

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where

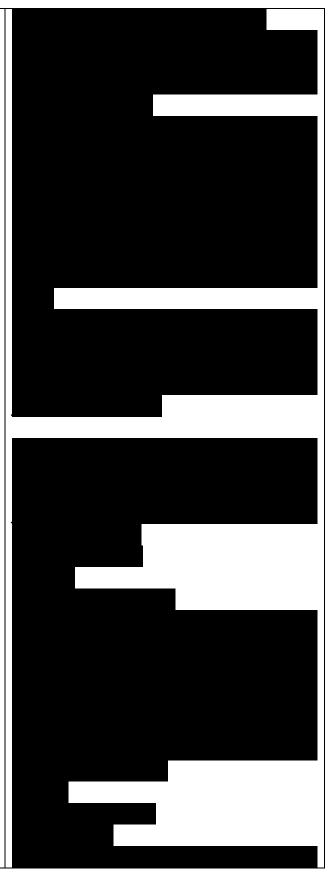
Proof. Let $j \in \{1,2, ..., [|] - 1\}$. Because n - j > |, the statement "at least a subset with n - j elements" is equivalent to "exactly a subset with n - j elements". n - j elements from n elements can be selected in,,,, ways. But the remaining j elements can be partitioned in B(j) ways. Hence.....

But

 $n]_/if n is ever,$

 $U\; 2 \; \backslash \; if \; n \; is \; odd$

Therefore, if n is odd, we have n - [|] >



, hence If n is even, we cannot partition the remaining | elements unconditionally, because when the set is partitioned in two subsets with | elements, \ ^ ^ ^ partitions are counted twice and therefore in this case, we have Example 2.1. Find the number of partitions of a set with 5 elements in which there exists exactly a subset with (a) 4 elements: (b) 3 elements. Example 2.2 . Find the number of partitions of a 4-set with at least a subset

with 2 elements (B(4|2)).

Solution. Using Lemma2.1, we have

These 9 partitions are as follows:

Now, we state two other lemmas about the number of partitions of a set with n elements in which all subsets have m elements. We indicate this number by Pm(n).

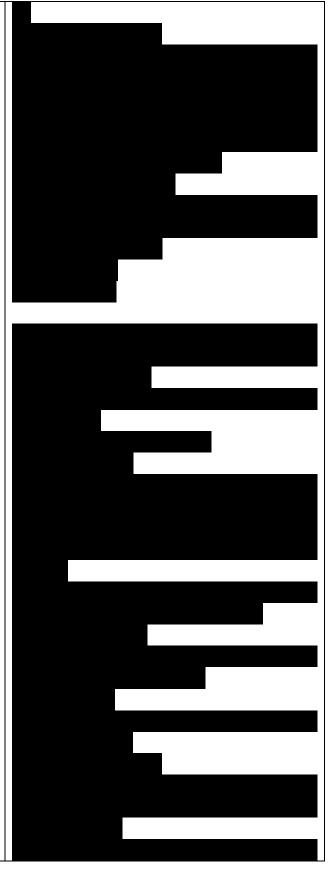
Lemma 2.2. If mln(i.e. there exist a natural number k such that n=km), then

Proof. The number of ways that we can distribute n elements to — subsets.

such that in every subset we have m elements, is

definition Hence, by using multinomial coefficients, this lemma is proved. □

Lemma 2.3. Let P'm(n) indicate the



number of partitions of an n-set to maximal number of subsets with m elements. We have

Proof. m[$^{\}$ is the greatest multiple of m that is less than or equal to n. These m[$^{\}$ elements are selected in $^{\}$ ways. But the remaining (n — to[$^{\}$]) elements must be partitioned arbitrarily. Thus, the relation is proved. \Box

Proof. The number of ways in which m elements can be distributed to j subsets, with numbers n1,n2, is

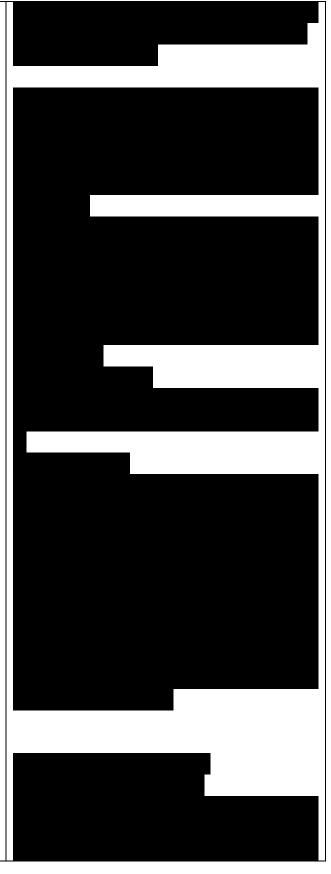
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But the number of elements of these i subsets isn't still m. Therefore n — m of the remaining elements must partitioned to i + 1 subsets, in numbers m —n1,...,m—nj, and finally n—jm. This number is ni=1 ($n km + ^1=0 ni y$ m - nkJ n0 = 0 or (y |. Now the n—jm elements ym n1,m - n2,...,m - nj,n - jm J are remained that must be partitioned. Using Lemma 2.2 and the product axiom in combinatorial analysis, the relation is obtained.□

Theorem 2.1. If m\n, then

.....

Proof. Consider m special elements of the set {1. 2. ...,n}, for instance These m elements can be distributed in ^



arbitrary subsets. Consider a case that m elements in j subsets are distributed in numbers n1,n2...,nj, j=1,2,YH=i ni=m• But the number of these cases is $i'>m|ri1,ri2,...,rij(^)$. Now by using Lemma 2.4 and summing over j, j=1,2,...,-, the proof is completed. \Box

Example 2.3 . Find the number of partitions of set {1. 2..... 9} in subsets vnth 3 elements,

- (a) without any condition;
- (b) 3 elements 1, 2, 3 are in 3 different subsets.

Solution. (a) Using Lemma 2.2, we have

(b) The number is P3\1t1,1(9). Now, Using Lemma 2.4, we have

Theorem 2.2. If n > 1, then

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Proof. By above lemmas, the proof is obvious. \Box

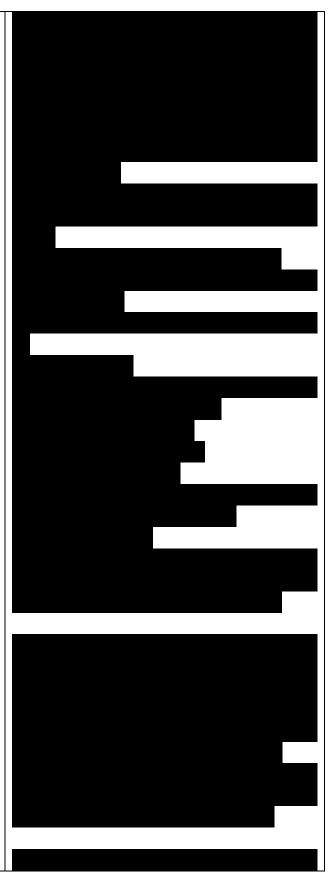
3 Partition function

Definition 3.1 . A partition of a natural number n is any non-increasing sequence of natural numbers whose sum is n.

In this section, we state some lemmas and theorems about P(n), the number of partitions of natural number n. By convention, we agree that P(0) = 1. It can be shown that P(1) = 1, P(2) = 2, P(3) = 3, P(4) = 5 and so on.

Lemma 3.1 . Let P(n|1, 2,...,m) be the number of partitions of a natural number n, such that each summand is at most m. Then.....

Proof. It is obvious that if m > n, then



condition "at most m" has not any restriction. Therefore in this case, we have P(n|1, 2,...,m) = P(n). Now, let m < n. In this case the greatest summand is m. Because we can arrange summands from left to right, non-increasingly, if in a partition we have summand m, the remaining number (n - m), must be partitioned; but not arbitrarily. This number must be partitioned such that the greatest summand of this partition is m. These numbers are P(n - m|1, 2,...,m).

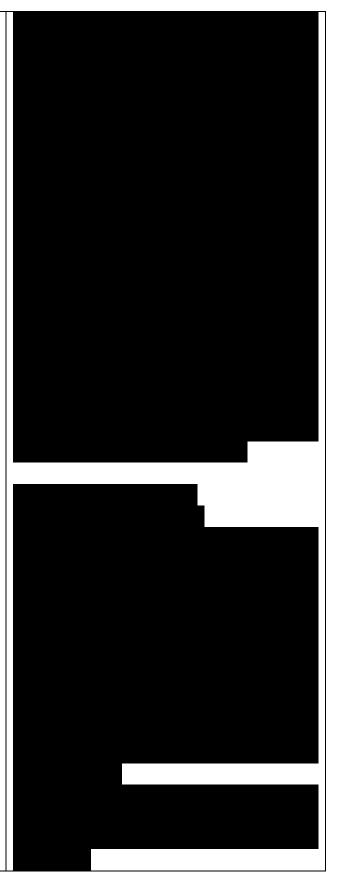
If we have summand m-1, the number n-(m-1) must be partitioned such that any summand is not greater than m-1. This number is $P(n-(m-1)\backslash 1. 2.... m-1)$. Continuing this method until the first summand in left partition is 1, and summing on the number of all cases, the relation will be obtained. \square

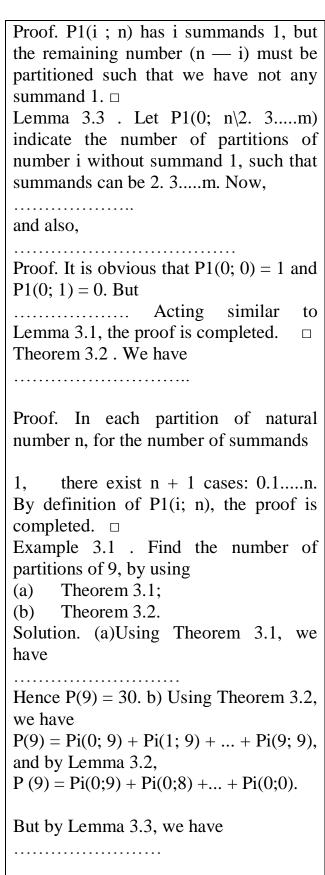
Theorem 3.1. We have

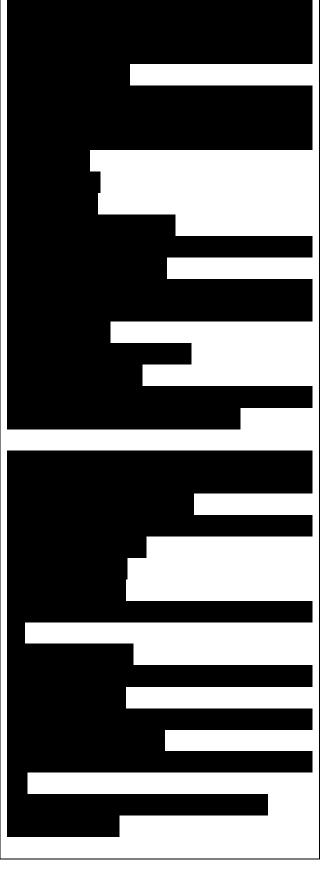
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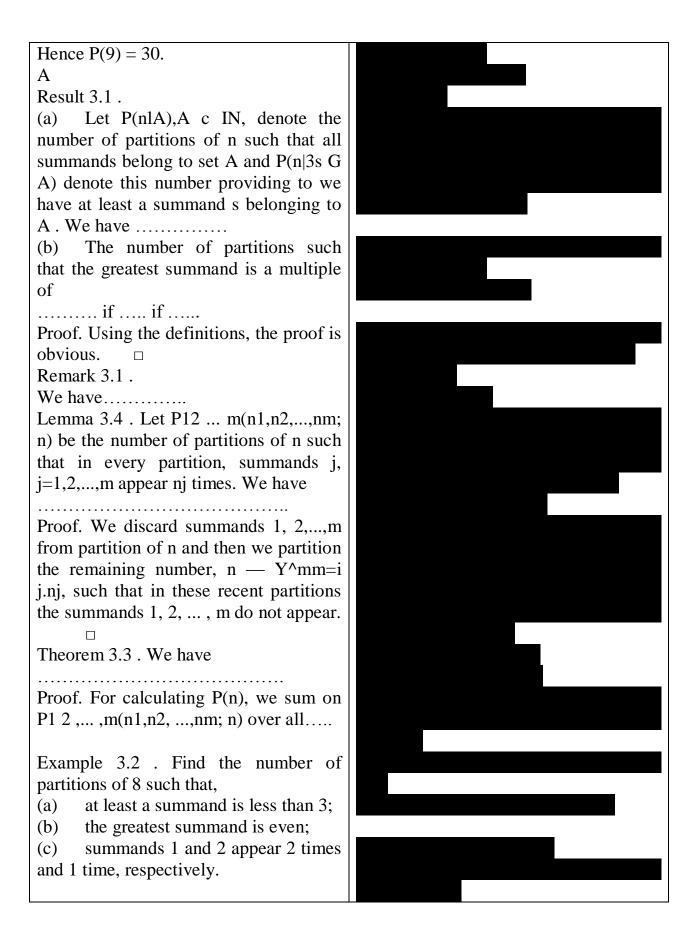
Proof. It is stated that P(0) = P(1) = 1 and P(2) = 2. Now, let n > 3. If the first summand is n - 1, then there is one case. Hence in general, if the first summand is n - i, i = 0. 1.n - 1, the remaining number i must be partitioned. Of course any summand can not be greater than n - i. But these numbers are $P(i \setminus 1, 2,, n - i)$. By summing on all cases, the proof is completed. \square

Lemma 3.2. Let P1(i; n) be the number of partitions of a natural number n such that in every partition we have i summands 1, exactly. Now.......









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Solution. (a) Using Result 3.1 part (a),	
we have	
$P(8 \mid a \mid b \mid a \mid b \mid b \mid b \mid b \mid b \mid b \mid b$	
P(8) — P(813, 4,, 8).	
But by Lemma 3.1, we have	
•	
(b) By Result 3.1 part	
(b),The greatest summand is	
even	
(c) The desired number is P1)2(2,1;	
8). By Lemma 3.4, We have	
This partition is $8 = 4 + 2 + 1 + 1$. A	